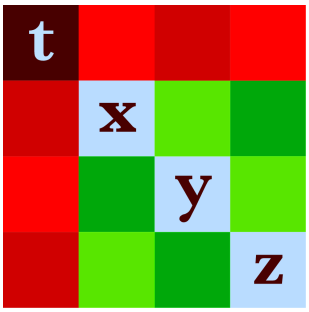

Substantive Waves - The Atomic Electrons

This is a working development document, liable to change without notice.

Tony Booth 16 March 2014

Summary:

An account of electrodynamics is given as wave solutions in a causal field model. The uncertainties explicit in conventional probabilistic quantum mechanics are here induced by inherent phase indeterminacies, recognising all observers as participants in a correlative process over a universal stochastic wave field. The existence of quantal behaviour with its associated attractor states is shown to emerge from joint solution of a pair of coupled equations (Maxwell and Klein-Gordon) that are individually familiar. Combined in this way their behaviour is crucially extended.

This approach transcends the dichotomous wave-particle duality inherent in the conventional Lagrangian models. It accounts for intrinsic spin in terms of factor functions in wave topologies ... unique to this sort of model. It offers better comprehension of this puzzling effect and of other inexplicable features of the conventional theory. In addition the model promises improved computational efficiency because it can replace the chaotic nature of the many body collisional problem with a multidimensional relaxation problem in smooth functions.

> The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them.
>
> Sir William Lawrence Bragg
> Nobel Prize for X-ray crystallography

Contents

1 A Continuum Model **5**
- 1.1 Motivations in Engineering . . . 5
- 1.2 Beginnings in Kinetic Dynamics . . . 6
- 1.3 Transcending the Newtonian Gambit . . . 9
- 1.4 Prospect of a Fully Local Model . . . 12
- 1.5 Schrödinger, Dirac and the Klein-Gordon Equation . . . 12
- 1.6 The Nature of Scalar Matter Waves . . . 13
- 1.7 A Closed Causal System . . . 15

2 Charge and Current Origination **17**
- 2.1 Intensity, Frequency, Charge and Energy . . . 17
- 2.2 Charge and Current Space-time Densities . . . 21
- 2.3 Charge Expressed in Frequency Domain . . . 22
- 2.4 About the Hilbert Transform Commutator . . . 24
- 2.5 Electromagnetic Propagation . . . 26
- 2.6 Interactions and Spectral Regimes . . . 30

3 The Matter Wave Equation **33**
- 3.1 Units and Variables . . . 33
- 3.2 The Modulated Klein-Gordon Equation . . . 34
- 3.3 Coupling Coefficients . . . 38
- 3.4 Frequency Domain - Modulated Helmholtz Equation . . . 42

4 Identifying the Quantal Ripple **47**
- 4.1 Spheroidal Mode Equation . . . 47
- 4.2 Separation of the Ripple Factor . . . 48
- 4.3 The Residual Modal Equation . . . 50
- 4.4 Orbital Magnetics . . . 50

5 Charge Ripple Integration **52**
- 5.1 Separating Structures in the Matter Wave Field . . . 52
- 5.2 Substituting Charge Density . . . 55
- 5.3 Spin/Ripple in Spherical Coordinates . . . 56
- 5.4 Convergence of Integrals . . . 60

6 Quantization **62**
- 6.1 Ripple Charge Invariance . . . 62
- 6.2 A One Dimensional Test of Topology . . . 62
- 6.3 Pauli Exclusion Principle . . . 65

7 Phenomena **66**
- 7.1 The Critical Evaluation of this Wave Model . . . 66
- 7.2 Lorentz Accelerative Forces . . . 66
- 7.3 Orbital Magnetic Moments . . . 67
- 7.4 Normal Zeeman Effect . . . 68
- 7.5 Anomalous Zeeman Effect . . . 69
- 7.6 Stark Effect . . . 69
- 7.7 Stern-Gerlach Effect . . . 69
- 7.8 Wherefore Magnetic Monopoles? . . . 71
- 7.9 Electromagnetic Emission and Absorption . . . 71

7.10 State Transition Selection Rules . . . 72
7.11 Aharonov-Bohm Effects . . . 72

8 Prospects and Further Work **74**
8.1 Feasibility . . . 74
8.2 Proving Task . . . 75

9 Acknowledgements **75**

Appendices **76**

A A Remarkable Smoothness **76**
A.1 Freedom from Lagrangian Process Noise . . . 76
A.2 Modulation of Coefficients . . . 76
A.3 The Natural Suppression of Up-conversion . . . 77
A.4 Avoiding Dichotomous Duality . . . 78

B Intrinsic Spin **79**
B.1 The Necessity . . . 79
B.2 Spin Factor Structure . . . 79

C Existence of Electron Eigenmodes **82**
C.1 Terms in the Mode-wave Differential Operator . . . 82
C.2 The Finite Balance in Oscillation Frequency . . . 82

D Inter-Modal Couplings **84**
D.1 Local Interaction Structure . . . 84
D.2 Precessions . . . 84

E Coefficient Matrix Structure **85**
E.1 Unitarity of the Coefficient Matrix . . . 85
E.2 Finding Stable Solutions . . . 86
E.3 Universal Diamagnetism . . . 87

F Wherefore Discreteness? **88**
F.1 BKS Debate - the Photon . . . 88
F.2 Complementarity . . . 88

G A Meta-model **90**
G.1 Stochastic Substrate Processes . . . 90
G.2 The Substrate Scattering Interaction . . . 90
G.3 Rationale for Charge/Current Origination . . . 91
G.4 Pauli Exclusion and the Fermi Distribution . . . 92

H "Azimuthal" Confusion **93**

References

Atomic Electrons as Substantive Waves

> A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.
>
> Max Planck, 1858 – 1947

1 A Continuum Model

1.1 Motivations in Engineering

In the face of the challenges to comprehension in modern physics we here investigate whether matter can be modeled on the substantive basis of waves in a continuum. We set out to build an all continua model along with the usual interacting fields like gravity and electromagnetism. Quantum phenomena then need to be emergent in the causal behaviour of the model without reliance upon any direct mathematically artificial discreteness.

Discussion of if and where the rational limits to such a possibility exist is not widely available. The rich history of the search for continuum models in physics is discussed by Alexander Unzicker [AU2000], but no clear conclusion is reached. Yet naive perception of discrete entities and events is not sufficient evidence for their existence as fundamental because soliton processes can in principle underlie any such appearances. So until we can prove otherwise we must continue to wonder whether such an approach to modeling might also explain some of the current puzzles in physics.

The motivation for this work also has prosaic and less speculative origins. It arises from challenges to engineering comprehension and modeling in photonic and other typically microscopic systems involving atoms, molecules and crystals.

A number of issues contribute to the latter problem, and amongst them is a desire to remain sympathetic to the kinds of mental conception and analysis that are more commonly used by engineers for continuous systems ... a preference for locally causal mechanism, and a desire to avoid the fractures brought about by treating mass bearing particles or densities in a Lagrangian system as fundamental when clearly from the physical evidence wave fields in a continuum must also always be involved.

Comparison with Current Quantum Mechanics

So in the approach here we shall not treat elemental particles bearing mass as fundamental. They must instead emerge as solitons under a model based directly on causation amongst field amplitudes. The quadratic (sesquilinear) products arising in this model then produce, as densities, the usual substantial properties of mass and charge and thereby the associated manifestations interpretable as momentum and force.

Furthermore we take issue over the battle in 1920s ... Einstein, Bohr, Planck, BKS, in which photoelectric and Compton effects drove the conclusion seeing the photon as essential. Contrary to their conclusion we here deny the use of the photon, using instead

only a continuum expression for electromagnetism. An outline account opposing necessity of the photon to account for photoelectric and Compton effects is given in appendix F on page88.

Notwithstanding the pragmatism of the original objectives and way of thinking the pursuit of this model has unexpectedly yielded more and more cases where it generates exactly the same phenomena as are encapsulated in what is the current quantum mechanics canon. However there remains a task in quantitative and detailed proof that it indeed matches or transcends that quantum mechanics.

It does not so far show signs of deviation from the actual phenomena that are predicted in probabilistic quantum mechanics. But more significantly it offers a different way of thinking of how they arise, and it goes further with those ideas. So perhaps it is most interesting to see this model as an account in underlying causal process of why individual measurements must in any case appear under the probabilistic kind of laws that we currently take as the whole picture.

This implies that, contrary to current ideas, hidden variables can account for quantum mechanical processes. But this will only be true so long as we do not always claim the absolute existence of specific entities (particularly photons) in terms of the prior measurements. It explains how measurements take on an inherently discrete nature, often without producing absolute evidence of existence of individual entities. To do this we model an underlying causal continuum world that is self-consistent and show that it is inherently only imperfectly observable ... see notes regarding complementarity in appendix F.2 on page88.

This elemental approach yields a model with a basis of fewer precepts than needed in quantum mechanics. This becomes most pronounced when studied in terms of an underlying stochastic field, and it then also offers a remarkable accommodation of cosmological phenomena. However that aspect will not be developed within this essay, and we confine our interest here to an upper level of the model dealing with what we know as electrons and their structures in atoms, molecules and crystals.

After a discursive introduction in section 1 the essay proceeds to summarize the model in an essentially mathematical way, guided and justified by physical explanations.

1.2 Beginnings in Kinetic Dynamics

The twentieth century development of quantum mechanics brought an intrusion of basic concerns beyond the Newtonian origins in the kinetic aspects of mass, force and energy. These new concerns involved discreteness of observation events and dependence upon underlying wave amplitudes. These amplitudes were then treated as a basis upon which complex quadratic (sesquilinear) norms yield probabilities of detection of elemental quanta conveying states of matter that are still conceived in the earlier kinetic forms treated by Lagrangian and Hamiltonian methods.

In these Newtonian derived models a path extremum nature is imposed as a precept using ideas like Hamilton's principle or Fermat's principle as primal. (See work by Erwin Schrödinger, for example his Nobel Prize Speech describing his version of Wave Mechanics

[ES1933].) The coefficients in Schrödinger's wave equation are contrived in terms of the kinetic parameters of discrete bodies in such a way as to yield wave parameters associated with that kinetic framework.

This procedure of kinetic formulation of wave equations has to be modified to suit different situations. It favours the Newtonian approximation ignoring relativistic behaviour. It becomes uncomfortably abstruse in cases where the Newtonian kinetic parameters are not naturally simple. The latter is increasingly common in modern pursuits such as photoelectronics and molecular analysis and then leaves this earlier conceptual basis with problems of comprehension.

Bypassing the Lagrangian Approach

Seeking means to an intuitive grasp the model is here set up with causal wave mechanism at its fundamental level. The conventional features of classical physics can then still, when conveniently applicable, be seen to appear as naturally emergent within the system kinematics. This achieves a more complete yet sparse set of precepts that determine the space-time continuum and its local differential wave constraints. Newer kinds of problems then have the desired simpler basis and the earlier problems can be handled in the conventional way, seen as an approximation to the new model.

The path followed here is reminiscent of the original steps taken by P.A.M.Dirac in using the Klein-Gordon equation. However, whilst Dirac developed a square root in order to focus on kinetic parameters, here the second order differential form of the equation is kept and developed onward into a substantive wave model in its own right. Those kinetically based results can then, if needed, be recovered from the wave solutions, but additionally other kinds of uses for the model and its concepts arise too.

There is a broad similarity, in both its purpose and in the use of fields, to the Density Functional Theory and the Kohn-Sham equations used in electron based molecular modeling [KC2006], but the approach here goes further and does not rely upon the conventional Lagrangian particulate model for its origins or support. Instead the wave model is set up in the manner of an Eulerian system as used for the fluid model by velocities at fixed coordinates (though not here actually as a fluid), relativistic from the outset, and without any allusion to an underlying particle model.

A Cleaner Basis in Sesquilinear Process

Elements with mass are quadratic (sesquilinear) in any underlying waves, and therefore do not offer a proper basis of explanation in the full range of the linear amplitude dynamics. The model proposed here is conceived directly in the complex waves. It accounts for classical physics in terms of a lower level model that is more general, and is also, once grasped, more simple in principle than current quantum physics.

The original mission was to build pure field models, approximate as necessary, to meet those needs, or else to clarify exactly where the road to such things was blocked. Some interesting structures have emerged, and in spite of that limited objective the study has uncovered prospects much more far reaching. The electrodynamics aspects will be outlined here, and just a mention will be made of those further possibilities.

The approach brings out a little recognized feature of an extreme temporal smoothness that is possible in certain quadratic (those called "sesquilinear") intensity functions of complex wave equation eigen-solutions. There is no equivalent to this in any model based on flying particles. This effect accounts for why electron fields do not radiate in their equilibrium states, and how there can be freedom from the "ultraviolet catastrophe" of up-conversion mixing of superposed electron fields whilst producing the down-conversion we call fluorescence. The usual particle based models encounter difficulties, demanding conceptual extension of the model, on both of these scores. Appendices A.3 and G offer an outline explanation of this idea.

Interacting Sub-systems
In the study of wave systems at large, substantial amounts of work have been done on nonlinear field equations with solutions of soliton form ... for example see Alwyn Scott's book [AS2003]. The approach there is normally to consider a single equation containing the necessary partial differential, plus quadratic or cubic nonlinear relationships. A comparable approach exemplifies the modeling of certain fundamental particles as "Skyrmions" as indicated by S.Wong in [SW2002].

By contrast the model considered here uses the interaction of sub-systems represented by two equations, both linear, but having the intensities of one driving the other whilst this latter then modulates the coefficients of the first. That gives rise to a cubic feedback loop operating to modulate, at lower frequencies of Rabi fluctuations, the structure of the other part of the system operating at high frequencies near to the Compton frequency. This high frequency system, it is the electron matter field, is maintained in and perturbed from its steady oscillatory state by the relatively low frequency electromagnetic sub-system. So there are good opportunities for approximation in reaching a picture of the respective solutions.

These two equations correspond respectively to descriptions of electronic matter (de Broglie-like) and electromagnetic (Maxwell) fields, so the resulting dynamics displays behaviour characteristic of natural phenomena. Beyond the relatively obvious requirements of kinetic (Lorentz forces) and spectral (luminous) phenomena, issues around Zeeman, Stark, Aharonov-Bohm and Stern-Gerlach effects have been used as key guiding checks upon its validity as a model.

Intrinsic Spin is Inherent
To remain true to its pure wave approach, this model demands that spin be recognized in topological terms as a wave function factor with explicit structure in the solution ... see Appendix B for expansion of this point.

Intrinsic spin turns out to be a necessary feature of all bound state solutions of the equations as formulated here if they are to exist at all. It appears in this model in structures that are described as "spinoids" and that can be accounted for as emergent in the solution space of a lower level stochastic wave process. This contrasts with current physics that again extends the maths and therefore also the fundamental precept base of the model to handle spin by the introduction of operators called "spinors".

Events, Quanta and State Attractors
In following the development of this model it has become apparent that the wave solutions constitute a form of discrete attractor system with spontaneous quantization properties. This essay describes the model to a point where the process of that emergent kind of quantization is described. This is markedly different from the current mainstream physics where the notions of quantisation are introduced as precepts in the mathematical structure ... not as emergent from the inherent causal nature of the field model.

Here particles are only at most emergent as solitons. Energy quanta, though not the exclusive form of dynamical behaviour, are essential artifacts of interactions involving attractor state changes of matter. Thus these quanta inevitably place ultimate limits on any process of microscopic observation, at least through structure necessary in every amplifier [AB2004].

This is clearly a lower level of model than the Lagrangian one because it does not rely upon mass, force or momentum in its definition. These quantifiable features are all found amongst the emergent nature that is made evident when the participating processes of observation are set up using necessary definition of partitions (e.g. observer and observed) within the system. (See work by this author regarding observation [AB2004].) We may associate the notions of energy and charge etc. with functions of the wave, but the kinetic variables associated with them in classical mechanics do not form part of the definition of the wave model. Thus the wave kinematics can be analysed alone, allowing kinetics involving mass, force and momentum to be interpreted from the kinematic pictures of interaction, observation and measurement. A clear distinction is visible between these levels.

One might ask whether the model described here is classical or not. Well in the sense that it is built from no more than locally causal rules (with the exception of a certain issue of Hilbert transforms hereunder) it appears to be classical. But when it is recognised that it does not depend upon mass, force and inertia as fundamental at the outset it then appears not to be classical. So perhaps we must not be restricted by this conventional question.

1.3 Transcending the Newtonian Gambit

Isaac Newton Extends the Original Paradigm
Three hundred years ago Newton inherited the mechanics of bodies interacting by direct mutual contact ... a locally contingent form of physics. But he observed celestial orbits and read such as Kepler, and consequently decided that to accommodate the evident additional effects further terms were necessary in the dynamical description.

To handle the nature of mechanical accelerative influence at a distance he introduced the notion of a field that we call the gravity field. He expressed compactly the nature of that field in terms of how matter generates it and how it in turn influences matter.

Thus he combined the prior paradigm of directly contingent mechanics with a new paradigm wherein influences are potentially present throughout space but that can only

produce real effect at points where matter is positioned within that spatial field. This latter is a kind of abstract emergence. You can see this particularly if you treat the influence as a force, because the magnitude of that force depends upon a property (we call it mass) of the object influenced. This latter dependence can be expunged from the model by insisting that the field only produces an accelerative tendency ... however, restriction to this potentially simplifying approach has not generally been stressed in the subsequent development of physics.

The form of post Newtonian classical physics is entirely based upon this disjoint kind of model. The resulting hybrid system is brought together in its most complete form as Lagrangian mechanics.

The introduction of electromagnetic influences has subsequently been developed on just that same basis of fields inducing accelerative tendency in matter and talked of as "force fields". The transition to quantum mechanics is also reasoned starting on a basis of that disjunction, with various modeling devices then added to correct its ill effects.

Thus the model of mechanics bequeathed to us by Newton contained two differently conceived parts, one constructed on commonly cognate material entities and their direct local interactions and the other formed as fields over the space-time continuum and mediating interactions manifested only in those original material entities. At the junction between the two schemes of modeling there arises the need for special additional primitive features to contrive an approach to completeness or closure of the combined model.

Seeking to Escape the Dichotomy

Such disjunction in model building tends to lead to substantial complexity, and has done so in the physics developed this way since Newton. Rom Harre [RH2004] introduces what he calls the three "Realms" of modeling. These include the two poles of modeling that are respectively based upon familiar entity elements (usually spatially localised) in one case and abstract propensities (often distributed or acting remotely) in the other.

The question then arises whether we can find a unified model that avoids that disjunction and its confusing effects. Since introduction of the gravity and electromagnetic fields seems to be the best way to handle these more recent needs of the model we are forced to consider whether a unification might be achieved by also shifting the matter paradigm to a field model.

This appears to be possible. We may remove the Lagrangian scheme from playing a fundamental rôle and begin instead with a pure continuum field model using variables that are functions of the Minkowski space-time. By choosing only the simplest forms of partial differential wave equations that produce solutions corresponding to natural phenomena the picture develops rather directly. It is necessary to replace material particles with fields as our set of conceptual elements, and the soliton is then essential as the notion of a localised persisting entity. That change is possible, and for at least certain classes of work is practical.

An immediately appealing feature of this approach is that the fundamental structures built in this way are easily made compliant with special relativity, and therefore remain so

in any combinations however complicated. Also there is no need to start with any kind of classical mechanics as distinct from quantum mechanics. Classical mechanics can instead be viewed merely as an approximation suitable for analysis of the mechanics of certain kinds of large scale systems. Thus for the purposes of modern requirements this kind of model can provide a direct and intuitive comprehension of phenomena that conventionally are regarded as advanced, abstract, opaquely mathematical, and sometimes even described as weird.

Amplitudes and their Sesquilinear Products

We may recognise a distinction between the pre-Newtonian part of the model where mass, energy and charge are handled directly, compared to the more recently introduced fields following Newton that have the nature of the amplitudes of spatio-temporally distributed functions. For such amplitude fields any associated variables like mass, energy and charge are not to be regarded as fundamental but instead must be derived as quadratic densities on those amplitudes. Thus we seek to unify by additionally confining the fundamental definitions of matter in this way to a similar kind of amplitude field.

It turns out most convenient to use complex scalar valued functions to represent wave amplitudes for matter. Then the attributes of mass, energy and charge arise as densities of bilinear product functions (i.e. sesquilinear functions) of the amplitudes.

In this domain of expression universally founded upon field amplitudes the fundamental equations quite naturally take forms entirely compliant with special relativity. The sesquilinear formulation introduces the commutator structures that produce behaviour characteristic of the naturally observed quantum mechanics. The closure of causal form through linking the coupling fields with the matter fields introduces cubic structure in the overall system, in spite of the linearity of the steady state forms of the individual equations. That cubic structure then supports emergence of wave solutions with stable soliton form for matter, and these solitonic solutions, when recognised as formed on a stochastic under-model, become inherently quantised in charge.

In the conventional quantum theory physicists use a special meaning for the term "Hilbert space". It has $6N$ dimensions for handling superposition of the probabilistic wave functions of N disjoint particulate entities. This is not needed here. It is replaced by a simple linear summation of superposed matter amplitude fields on the 3+1 dimensional Minkowski space-time. Integrals of sesquilinear functions then provide the rest of the structure quite naturally.

That then is already a better unified model than available from current physics literature. In particular it leads to many detailed simplifications in the expression of quantum mechanical processes. But it offers paths to even greater unification in a cosmology based on an underlying stochastic process with Planck bandwidth, realised on a single universal complex scalar wave field. Furthermore it has promise for modeling the dynamics of atomic nuclear processes as a seamless extension of wave solutions beyond the lower order ones that constitute the electrodynamical model.

1.4 Prospect of a Fully Local Model

Examples of natural non-local phenomena are often cited in such as Young's two slit experiment, and the various non-local optical demonstrations such as the Alain Aspect experiments [AA1982] and long distance entanglement as used in secure code transmission [NL2004] [FD2007].

We develop here a continuum model in which all causal processes rely only on local influences. Therefore these apparently non-local effects are then (notwithstanding John Bell's pronouncements in [JB1987]) to be accounted for by processes within the behaviour of the system we so describe. Those accounts are being developed separately from this essay.

There is an exception to strict locality at one point in this model, namely the use of a Hilbert transform, but that also is corrected in the fully developed model with its unifying stochastic underlayer. That detail is presented in brief outline in appendices A.3 and G in this essay, but remains to be fully presented elsewhere.

Preventing non-locality from forming part of the model in this way improves flexibility of model applications and developments. It is particularly helpful from the point of view of comprehension in engineering design. In this respect this essay carries further the motivation of T.W.Marshall to justify general use of a non-quantal electromagnetic wave [TM2002] [TMa1997].

1.5 Schrödinger, Dirac and the Klein-Gordon Equation

The conventional Schrödinger and Dirac equations do not address the issues at the core of the model described here. Because they are framed in the characteristic properties of a discrete entity (a particle), the values of energy and momentum treated by these conventional models are intensive. But in order to build the picture more fundamentally it is necessary to conceive of extensive variables so that the structures within such a particle, when regarded as a soliton, can be studied in parts.

There might appear to be similarities between the model described here and the field equation developed by Dirac in the 1930s. However the similarities are only superficial. The approach is in fact substantially different.

Dirac derived from the Klein-Gordon equation a relativistically valid result in a first differential order format. It yields a form of rotational invariance that is commonly interpreted as a representation of the intrinsic spin phenomena with a half order rotational operator (called a "spinor") in a double cover of the physical rotational space. It involves expressing the field variable as a vector of four components. In keeping with current theories of quantum mechanics, the wave field is conceived as an amplitude underlying a spatial probability distribution of the position or momentum at which the respective electron can be found.

In the model proposed here we also start from the Klein-Gordon equation but instead seek a substantive construction ... one in which the field represents the substance of the electrons around the atom and models their physical behaviour. Also our model does not

assign negative frequency to the domain of anti-matter.

We keep the second order form of the relativistically invariant Klein-Gordon equation and apply it to a complex scalar field variable. We first recognise a restricted set of field function forms (see below "bi-cisoids" towards the end of section 2.1) that can solve it. It happens that those functions require a structure composed of four cisoid components, but they are additive to produce the scalar variable. The interlocking of these four cisoid components is not as independent eigenfunctions but rather as controlled by an underlying stochastic process enabling existence of such solutions. By this means we arrive at a half order rotational effect in an abstract 4-space, leaving the nature of physical rotative transformations unaltered. The element factor of scalar field wave solution with this property we here call a "spinoid". In this model intrinsic spin is a topological state essential to existence of the solutions, not a dependence upon physical rotation or angular momentum. See appendix B for further notes regarding this view of intrinsic spin.

Here we form integrals over a continuum so as to arrive at the structure of a distinct entity, but only in the form of a soliton, never as a point particle. Instead of considering just the entire energy or charge of a particle we work with the intensity in a wave field, and show how that can coalesce into a soliton with spatial integrals producing its characteristic values of charge, magnetic moment and energy. Furthermore we are free in the model described here to maintain a full view of the electronic matter fields and to take advantage of their linear properties in the short term under additive superposition. (See further notes on this point under section 3.2 below.)

The mission is thereafter to demonstrate and to check rigorously how far this compact and basically simple system goes as a valid model of natural physics.

1.6 The Nature of Scalar Matter Waves

We begin by basing a wave structure for matter upon a matrix modulated version of the Klein-Gordon equation (for its structure see equation (3.1) below). We use this because it is the simplest scheme with the necessary relativistic invariance, and that also gives rise to the dispersive wave velocity that is an essential feature of the de Broglie kind of matter waves that evidently characterise certain important aspects of behaviour of matter. For most purposes of analysis here this can be transformed into the temporal frequency domain where it is represented by a corresponding matrix modulated Helmholtz equation.

Problems with de Broglie's Wave

It was posited by de Broglie, and is still recognized in modern physics, that any matter particle behaves as though it depends upon rectilinear waves with a wave vector corresponding proportionately to the momentum of the particle, a wave number (spatial frequency) that is the magnitude of that wave vector and a temporal frequency determined by energy.

Though the effects are demonstrated in cases from electron beams up to certain forms of atom interferometry experiments, the de Broglie relationship does not constitute a complete model. It does not, for instance in these atom or molecule interferometers, explain in any exact way the basis of interaction of transiting particles with the material of the fixed

gratings that are used to scatter them. As another example, there is no way to determine what might happen if a particle were to consist of two very weakly bound parts ... how weakly would they need to be bound before they changed their apparent de Broglie waves so as to accord with being two particles?

Relativistically Consistent Solitons
A much wider set of solutions based on solitons is brought about by generalising the Klein-Gordon or corresponding Helmholtz equations through using the electromagnetic field to determine the 4×4 matrix of coefficients embedded in the second order differential operator (generalised d'Alembertian). For those composite structures such as atoms and molecules containing many resonant parts each with individual eigenfrequency and wave vector these rules apply to the individual modes rather than to the aggregated composite particle. Internal stresses between separate components when in accelerative motion are then borne by the overall soliton state attractor mechanism inherent in the wave process.

This picture accounts for how matter interferometry experiments display effects corresponding to aggregated mass for the composite particles. We see that interactions between the fringe fields of the particle and the electronic eigenmodes of the grating produce the accelerative effects (that we may interpret as forces), and the large diffraction angles these would naturally produce are reduced in inverse proportion to the mass that cohesively forms the overall particle. Completion of that picture in detail goes beyond the content of this paper, but it offers a natural relativistic explanation, which is not so for the de Broglie proposal for behaviour based upon mere association of a plane wave with each overall composite particle.

To analyse atoms using this model we may solve in the static case for the localized form, in simple atomic cases typically spheroidal (but not a simple sphere). Then if required we may multiply each separate modal wave with its respective rectilinear wave factor corresponding to translative velocity. This maintains the proper form of invariance under Lorentz transformations.

Though we are in this way generalizing the wave concept introduced by de Broglie beyond the rectilinear factor, still the terms ψ-wave, matter wave and de Broglie wave will be used interchangeably herein.

Point Nucleus Approximations
For this initial portrayal of electronic wave dynamics the atomic nuclei will be treated as having point mass and charge at the requisite positions. For pure topological issues of the model of bound electrons that turns out to be less critically important than it is when working with the conventional Lagrangian model because here we are not dependent for topological structure upon an imposed Coulomb potential field. Instead it is electric field divergence (i.e. charge density) that determines the broad topology of superposed collections of harmonic matter waves, and does so up to the definition of quantal structure. This leaves the other effects of the electromagnetic field as quasi-conformal deformations of the wave mode shapes, sizes, energies and frequencies, and for future clarification, effects upon their stability.

1.7 A Closed Causal System

In order to have a closed loop of effects constituting a causal model for electrodynamics we shall first need to depict, in section 2, the origination of charge, current and electromagnetic field. This will support a cycle of causal influences whereby a system developed from the Klein-Gordon equation mediates the form of electron wave in a given coupling field and the model of charge origination developed in section 2, operating through electromagnetic propagation, returns the form of coupling field for the given electron wave. The causal cycle is thereby closed.

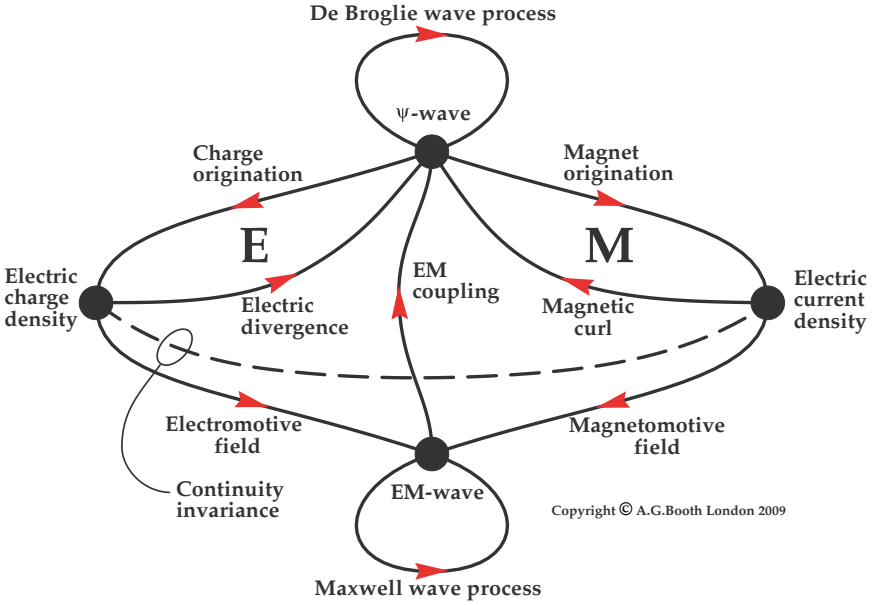

Figure 1.1: **Causal Linkages in the Wave Model**

Figure 1.1 shows the general disposition of causal linkages for the model. This type of diagram summarizes the fields and their interactions that make up the entire system.

Each nodal dot represents a variable that has a value at every point in the four dimensional space-time. The ψ-wave is complex scalar, the charge density is real scalar, the current density is real three-vector and the EM (electromagnetic) wave is generally thought of as a related pair of real three-vectors. The four-vector potential could be considered at this point, but in this model the scalar electric potential is not treated as fundamental in analysis of relativistic dynamics.

The four-space in which all these fields exist as wave functions is referred to as the Minkowski space to denote the way that one dimension, that associated with time, is treated differently (hyperbolic) from the other three positional space dimensions with their Euclidean geometric properties. This is the basis of special relativity in the model.

The inner loops marked E and M deal with the dynamics of charge and current regulation. Indeed, it is only the E loop that takes effect in determining the distinctions between the discrete attractor states of different ionic charge distributions. Magnetic quantization involving loop M is more subtle. The remaining causal paths at the lower part of the diagram are via electromagnetic fields. These latter effects are mainly observable as interactions between separate charge or current bearing entities. The processes supporting inter-atomic molecular bonding involve either or both of these sorts of causal systems, the upper loops dominating in what are called valence bonds and the lower loops in ionic bonds.

2 Charge and Current Origination

2.1 Intensity, Frequency, Charge and Energy

To make our closed causal model possible we must define how electric charge and current are manifested in terms of the de Broglie matter wave field. The coefficients of the Klein-Gordon matter wave equation (we shall use a matrix of these coefficients) are then influenced by the distributed form of the charge, and are therefore functions of space-time co-ordinates. Because of this we might refer to this form of the equation as a Modulated Klein-Gordon equation.

To connect up the system of coefficients in this way we first describe the rules governing how the electric and magnetic effects (charge and current) arise from the ψ-wave field. At the end of this section we shall then look at the nature of the related effects acting at a distance, and that will be in terms of the system of electromagnetic wave propagation.

To model the observed interactions of matter we may begin with an underlying process with the nature of scattering. The kind of scattering in which we are interested here is described conventionally in its outward effects in terms of the electromagnetic field. We can never directly perceive the electromagnetic field, but we can build its description by using interactions between material entities as evidence for its existence. We can do a similarly inferential thing to build our entire causal model.

What we refer to as charge and current can be characterised as the matter wave intensities that give rise to the scattering process ... these are the sources that drive the electromagnetic field. These sources form a vector with four elements and we may describe them as comprising a scalar value of charge density and a 3-vector of current density.

A full account of the scattering process would require a deeper model in the form of a stochastic wave system. Here we shall limit ourselves to an empirical determination based upon the directly observable characteristics of these interactions, but still going somewhat further as a unified model than is the case in the conventional quantum theory.

Sesquilinear Inner Products and Functions

The regulatory and control effects of the electromagnetic field are crucial to the way the model operates. However, in terms of the underlying matter waves the only way that matter can influence the electromagnetic field is via charges and currents, and these are always constructed from a certain kind of inner product of the matter wave amplitude variables. Given two such amplitude variable terms A and B at a given point in space-time, the element of this form of inner product is then $A^*.B$ where the star superfix denotes complex conjugation.

Functions that involve original variables exclusively via this kind of inner product of amplitudes are called "sesquilinear", and charges and currents are always so. Correlations between pairs of matter wave variables are also defined as averages over these kinds of products.

Further to this restriction on the class of wave functions involved we pay special attention to the cases where every element of matter wave modal structure is built with bi-cisoid

form.

Bi-cisoid Wave Functions

The matter waves ψ considered in this model are complex scalar, but are also restricted to a form that we shall call "bi-cisoid" denoted here by the set symbol $\mathbb{B}$. This description relates to the temporal derivative structure of the function. It says that with respect to time such functions can always be built from elements that are each the sum of two equal but opposite frequency cis functions. The cis function has the form $\text{cis}(\omega t) \equiv \cos(\omega t) + \jmath \sin(\omega t) \equiv \exp(\jmath\omega t) \equiv \epsilon^{\jmath\omega t}$. So a general bi-cisoid function $\psi \in \mathbb{B}$ can always be expressed as a sum of elements each with the form:

$$\psi(t, \mathbf{x}) = \psi_S(\mathbf{x}).\cos(\omega t + \phi) \quad \text{where } \psi \in \mathbb{B};\ \psi_S \in \mathbb{C};\ t, \omega, \phi \in \mathbb{R};\ 0 \leq \phi < 2\pi;\ \mathbf{x} \in \mathbb{R}^3$$

We can think of each such wave element itself as a function on the space-time coordinates that is a complex oscillation with a single constant temporal frequency throughout space and time (such an element is coherent) and where the values of the complex argument and the phase are functions of the space coordinates (not generally of time). It can always be treated as the sum of two equal magnitude counter rotating cisoid functions ... hence the name "bi-cisoid". Such a combined wave is complex without the full generality of a pair of independent cis functions. It offers properties in a complex wave description that are similar to a real scalar wave in having a complex argument that in the steady state is locally constant to a good approximation. Matter waves model well on this basis.

In a strict sense these bi-cisoid functions apply only to the exact steady state of matter. However, the perturbations from that exact form need only be relatively very small or slow to support the range of matter dynamics that we encounter practically. Thus we may expect that at most a first order term is all that would be needed to handle those perturbations.

For bi-cisoid functions (or distributions) with composite spectra (i.e. not simple line spectra) the same definition will apply in the sense that any variable can be composed of a sum (spectrally either discrete or continuous) of bi-cisoid element functions at individual frequencies.

These bi-cisoid functions have even symmetry of power density spectra with respect to the time dimension.

Use of these bi-cisoids is a convenient simplification here to represent functions in a unified picture of linearly superposed matter wave fields. This is distinctive in this model, and replaces the current conventional method requiring norms of individual electron waves to be evaluated and subsequently combined.

The bi-cisoid property arises as a result of a stochastic average formed at very high frequencies around the Planck frequency in the region of 10^{42}Hz and they can be formed over averaging periods that are short compared to anything we need to consider for electrodynamical issues. Because this required averaging period is short we are also free to use the notion in a comparably short term sense, allowing us to consider a bi-cisoid function with frequency structure that varies with time at all practical frequencies of evolution of the "steady" state of matter that we need for electrodynamics ... for current engineering purposes that might be up to say 10^{16}Hz.

Further Structure in Charge and Current
The strength of electromagnetic induction is based upon a 4-vector sesquilinear product between the scalar value of the matter wave field and its space-time four dimensional gradient. To that extent our model is similar to that used by Dirac as applied to the field solutions of the atomic wave equations of his and Schrödinger's work, and that aspect is now conventional. However to accommodate a wider range of phenomena a Hilbert transform is needed to shift that result to a real number, and a commutator form of that operation is needed to achieve symmetry (see equation (2.1) below).

By these means the natural phenomena of fluorescence combined with an absence of electronic radiative up-conversion (i.e. no ultraviolet catastrophe) can be directly accounted for in our model. And this is achieved with the added simplicity of an ordinary kind of additive superposition of matter wave fields in 3-space instead of the usual 6N dimensional configuration state space for N particles used in conventional quantum mechanics and in Lagrangian mechanics.

Adjustment of Differential Order
In the model underlying Schrödinger's equation the origin of charge depends upon intensity of the wave function, and the associated current is derived from a conjugate (i.e. sesquilinear) product of amplitude and spatial gradient of that same wave function. Here the symmetry of the model is improved, in a manner similar to that used by Dirac, by making the entire four-vector of charge and current come from a sesquilinear product of the wave amplitude and its 4-gradient. This has the effect of reducing the temporal (but not the spatial) differential order of the wave variable by one (i.e. using the integral with respect to time of Schrödinger's wave variable). Expressions based on wave intensities are therefore multiplied by $|\omega|$ for charge and ω^2 for energy densities, where ω is the angular frequency of the respective electron mode.

In many respects such changes of differential order of the primary variable have little effect on the model, but a consequent reintroduction of symmetry brings with it new opportunities for generalized structure of charge and current density fields. In particular it enables the introduction of the Hilbert transform, bringing greater smoothness of couplings and a more marked orthogonal independence of the eigenfunctions in the wave solutions. This way the smoothness extends remarkably to the electromagnetic fields produced by additively superposed modal wave functions. It is thereby made more realistic in avoiding both electron radiation in the bound steady state and the UV catastrophe effects of field intermodulations. A further (though limited) discussion of this can be found below in appendix G.3.

Spin and Signs of Frequencies
With this change from convention in the dynamical definition of the model it becomes natural (and for feasibility of topologically self consistent spin structure, indeed necessary) to employ both signs of frequency in the description of the electron field. Thus the Dirac

inspired convention of associating anti-matter with negative frequencies will not be used here, and instead alternative signs of a matter parity symbol ϖ (called "variant pi" or "varpi") will be applied to the modulus of frequency factor in the charge and current origination formulæ ... $\varpi = -1$ for negatively charged matter and $\varpi = +1$ for positive. For ordinary electron fields this symbol will sometimes just be omitted and where appropriate replaced by a fixed sign. The explanatory and analytical basis for this sign change can be found in the underlying stochastic field model, but beyond notes in appendix G.3 will not be developed in this essay.

Use of Hilbert Transforms

From the early stages of development of this model it was clear that some special kind of differential first order structure was required in order to introduce the nature of charge. The entry of the Planck constant multiplying frequency in the first degree as $E = \hbar\omega$ to produce a measure in energy for a given charge shows this up. Whereas a differentiation or an integration will introduce a first degree proportionality with frequency, the result always produces an imaginary variable. The charge relationship does not do that.

Seeking a simple way to model the required relationship pointed to the Hilbert transform ... a linear operator with the very characteristics needed. However it brought with it the inherent snag of having a necessary non-causal form ... it produces a result that has dependence upon sesquilinear inner products of separated points in time, including future points. Similar transforms are used in signal processing, but when ever time is involved they have to be delay shifted sufficiently far as to leave negligible dependence upon future input, so cannot provide immediate output of their result. In our model we are not generally in a position to do that.

In spite of this incompatibility with causal modeling the behaviour of the Hilbert transform showed itself otherwise to be very much true to the requirements. In particular it showed that it would model the electron in equilibrium as radiating no energy, and that it could produce freedom from up-conversion in interactions between superposed electrons whilst still delivering the kind of down conversion interaction that models fluorescence. Nothing simpler than this would appear to be capable of introducing the required form of field interactions to the model.

Justifying the Hilbert Transform

Extending the search for justification for this model construct suggested that the Hilbert transform might be our perception of the stochastic behaviour of an underlying system operating at much higher frequencies. After all, the current appearance of physics includes relationships suggesting the involvement of frequencies up to the Planck frequency around 10^{42}Hz.

The investigation has continued along these lines, and as a result has led to a cosmological view of the basis of the model. It has turned out that there are features of the behaviour described in General Relativity that tally with this view. In particular the Einsteinian light bending effects and, perhaps more directly, the Shapiro effect of

delay in electromagnetic signals passing through regions of reduced gravity potential, have corroborated such a model.

So the construct introduced as the simplest precept with the necessary behaviour at this main level of the electrodynamics model has fallen into position to become part of a substantially simplified under-layer field model that offers unification between matter, electromagnetics and gravity.

The Hilbert transform as introduced to the original main (i.e. upper) level model is therefore a precept at that level, and it introduces a non-causal mechanism as a stochastic approximation. This is the only non-causal relationship installed in this main level of the physics model. It is justified by its ability to comply with a deeper model in which no such non-causal relationship is introduced. However, that lower level stochastic model is itself then supported by a deterministic wave model with wave velocity much higher than the speed of light. This high speed of wave propagation is only detectable indirectly (as far as we yet know) as necessary in support of the overall attractively unified compact model that it allows here.

2.2 Charge and Current Space-time Densities

To comply with the Planck constant proportionality between (modulus of) frequency and observed energy we need a simple model whereby charge will be generated locally as dependent proportionally upon both intensity (i.e. squared magnitude) of ψ-field excitation and its (modulus of) frequency. The following model does this and also shows how the same field gives rise to a magnetizing vector in a related manner. The resulting electromagnetic field can then return as causal coefficients in the Klein-Gordon differential operator matrix.

Charge and current densities are obtained in terms of functions of matter wave intensities and gradients by the following expressions. The symbol $\boldsymbol{H}_t(.)$ denotes the time domain Hilbert transform. Using $\star$ to denote convolution, this is an integral transform of the form:

$$\boldsymbol{H}_t(f(t)) \;=\; -1/\pi \int_{\mathbb{R}} f(t+\tau)/\tau \; \mathrm{d}\tau \;=\; (1/t) \star f(t)/\pi \qquad t, \tau \in \mathbb{R};\; \boldsymbol{H}_t, f \in \mathbb{C}$$

It produces a $\pi/2$ lag for +ve and lead for −ve frequencies: $\boldsymbol{H}_t\left(\mathrm{cis}(\omega t)\right) = -\jmath\,\mathrm{sign}(\omega).\mathrm{cis}(\omega t)$

The complex conjugate of ψ is denoted by ψ^*. So using ϖ (called "varpi") to denote sign of charge and i, j to index modal field terms, real valued charge and current densities are then:

Electric charge scalar density **EHt**

$$q(t,\mathbf{x}) \;=\; \varpi \sum_{i,j} \left(\; \psi_i^*(t,\mathbf{x}).\boldsymbol{H}_t(\partial_t \psi_j(t,\mathbf{x})) - \boldsymbol{H}_t(\psi_i^*(t,\mathbf{x})).\partial_t \psi_j(t,\mathbf{x}) \; \right) \tag{2.1}$$

Electric current vector density

$$\mathbf{i}(t,\mathbf{x}) \;=\; \varpi \sum_{i,j} \left(\; \psi_i^*(t,\mathbf{x}).\boldsymbol{H}_t(\nabla \psi_j(t,\mathbf{x})) - \boldsymbol{H}_t(\psi_i^*(t,\mathbf{x})).\nabla \psi_j(t,\mathbf{x}) \; \right)$$

To do this compactly we use a four dimensional differentiator symbol $\lozenge$ (called "lozenge"). As is the case for the vector differentiator ∇ (called "nabla" or "del"), when applied to

a vector there is an implicit contraction of the result to produce a scalar (but not here). Using this symbol the above equations EHt can be combined in a four-vector for charge and current as:

$$\mathbf{q}(t,\mathbf{x}) \;=\; \varpi \sum_{i,j} \left(\; \psi_i^*(t,\mathbf{x}).\boldsymbol{H}_t(\Diamond\psi_j(t,\mathbf{x})) - \boldsymbol{H}_t(\psi_i^*(t,\mathbf{x})).\Diamond\psi_j(t,\mathbf{x}) \; \right)$$

The conventional physics assigns a value to charge by units that are 4π smaller than would naturally suit us here. Why did that happen, historically? Should we just multiply by 4π here to stay with convention, or could there be a better scheme than that?
This temporal Hilbert transform operates non-causally in time and is therefore not a fully qualified component of a locally causal model of the wave dynamics. It is valid because the averages that are the basis of the Hilbert transform are formed at very high frequencies indeed. More exact expressions of this involve a lower stochastic layer of the wave model that will not be developed in this essay beyond some notes about it in appendix G and in section 2.4 below. So the existence of a Hilbert transform, as introduced for the purpose of this level of the model, must be treated for the time being merely as a precept.

2.3 Charge Expressed in Frequency Domain

Separation of the handling of terms at different temporal frequencies is rather more intuitive and convenient in the one dimensional frequency domain (either as discrete frequencies or as Fourier or Laplace transforms) than it is with time and space together remaining as represented in a four dimensional partial differential equation. This frequency approach is particularly suitable for atomic systems because the Hilbert transform described above is simpler to handle in terms of individual frequencies. It is a fully valid approximation in the steady state (i.e. with very narrow frequency bands) and we may progress from that simple smooth case into higher degrees of perturbation (generally with broader frequency spectra).

This is in contrast to conventional physics where starting out with particles in the model spoils this natural sense of maximum smoothness in the simplest cases because of the indefinitely high values of derivatives of fields around such particles and the apparently irrelevant chaotic scattering effects that this introduces in the Lagrangian kinetic model. Actually, the evaluation of field strengths by inverse square law in the pure wave model bears a resemblance to the integration of inter-particle repulsions in a wave-probabilistic kinetic model (so why the exchange correlation hole? Is it specific to the DFT?). The differences are less pronounced than the inverse square relationship itself. Perhaps the cubic is the main difference. We must adjust this, and try to clarify the distinction between these paradigms ... embracing, if possible, both the DFT and the QFT models.

Thus the above equations EHt (2.1) for origination of charge and current can valuably be expressed in their corresponding temporal frequency domain form against generalized frequency $s = \sigma + \jmath\omega$, and for practically all purposes here we use $\sigma = 0$ to correspond to the "steady state" of oscillatory fluctuation at constant magnitude.

So using ϖ (called "varpi") to denote sign of charge and i, j to index modal field terms:

Electric charge scalar density **Echarge**

$$q(t,\mathbf{x}) \;=\; \varpi \sum_{i,j} (\text{sign}(\omega_i) + \text{sign}(\omega_j)).\text{cis}((\omega_j - \omega_i)t).\Psi_i^*(t,\mathbf{x}).\omega_j.\Psi_j(t,\mathbf{x}) \tag{2.2}$$

Electric current vector density

$$\mathbf{i}(t,\mathbf{x}) \;=\; -\varpi\jmath \sum_{i,j} (\text{sign}(\omega_i) + \text{sign}(\omega_j)).\text{cis}((\omega_j - \omega_i)t).\Psi_i^*(t,\mathbf{x}).\nabla\Psi_j(t,\mathbf{x})$$

To express this compactly we use a hybrid differentiator symbol consisting of one multiplier element s and a three-vector differentiator ∇ as $\Diamond_s \;\equiv\; \{s, \nabla\}$, and again, as for equation EHt (2.1) above, when applied to a vector this symbol conveys an implicit contraction to a scalar (but not here).

The purpose of the hybrid symbol is to simplify expression of the nature of steady state matter in which the wave system has a static factor for its spatial structure and a dynamic factor that is continuously oscillating temporally at many eigen-frequencies as the basis for the persistence of the matter. Using this symbol the above equations Echarge can be combined in a four-vector for charge/current density as:

$$\mathbf{q}(t,\mathbf{x}) \;=\; -\varpi\jmath \sum_{i,j} \Big(\; (\text{sign}(\omega_i) + \text{sign}(\omega_j)).\text{cis}((\omega_j - \omega_i)t).\Psi_i^*(t,\mathbf{x}).\Diamond_{\jmath\omega_j}\Psi_j(t,\mathbf{x}) \; \Big)$$

Whilst confining our use of frequencies only to the time dimension is related to convenience of expressions of persistently time-oscillatory fields, in contrast the Hilbert transforms and their corresponding frequency domain structures are installed as a fundamentally based description of interactions. Nevertheless the Hilbert transforms are also associated with the time dimension alone. We need to work with the stochastic under-model to cast light deductively on why the Hilbert transform should enter here, but that will not be dealt with here beyond an outline in appendix G and the notes following here below.

Building Matter Fields from Bi-cisoid Elements

The above expressions for charge and current are all in terms of temporal oscillations of complex variables. Let us call these "cisoid" functions on account of their temporal factor having the form $\text{cis}(\omega t + \phi) \equiv \epsilon^{\jmath(\omega t + \phi)}$. But this leads to some anomalous appearances such as the abrupt changes of value as a frequency passes through zero. These are not real effects and result from handling separately the opposite frequency parts of field terms that must always operate in pairs with opposite signs of temporal frequency.

So it is helpful to use expressions that combine these pairs into what will then be half as many modal terms, two pairs instead of four independent parts per bound electron, thereby showing no such anomalous features ... see "Bi-cisoid Wave Functions" page18. The modal frequency of the dynamic bi-cisoid (bicis) pair of elements of electron i can then be referred to by the signed frequency $\omega_{dt,i}$ or just ω_i of its twisted element, the sign being

its spin state, and the associated static pair of elements then needs no further means of identification.

The relative phase angles between bi-cisoid pairs i, j can be expressed as ϕ_{ij}, but the relative phase of the two elements of a bi-cisoid are merely determined by the complex argument of the electronic mode amplitude. The two bicis elements of the intrinsic spin factor structure of a single electron mode, one oscillatory (dynamic) and the other static (zero frequency), each have the same magnitude in ψ.

The following result of combining the two elements of each bicis function seems to avoid the use of non-causal (temporal future non-local) expressions. Is there an error here? Would this also happen if the restriction to bicis functions were made before the original point of deriving the charge density by Hilbert transforms? What would be the form of the bicis charge density expression done that way? Does it (can it) produce this same result hereunder? It appears that we may just be evaluating a stochastically influenced integral, presumably one that must inherently contain stochastic noise once the fact is appreciated.

Does the inclusion of the zero frequency bicis term again introduce non-causal form? If so then a full derivation in the first instance using all four spin factor elements might be the best bet.

Defining csvec $\equiv \{\cos, \jmath\sin, \jmath\sin, \jmath\sin\}$ as a 4-vector operation, the bicis form of equation Echarge (2.2) then becomes:

$$\mathbf{q}(t,\mathbf{x}) \;=\; -4\varpi\jmath \sum_{i,j} \text{csvec}((\omega_j - \omega_i)t + \phi_{ij}).\Psi_i^*(t,\mathbf{x}).\Diamond_{\jmath\omega_j}\Psi_j(t,\mathbf{x}) \tag{2.3}$$

It remains possible to group these bi-cisoid functions further so as to treat the four terms in each atomic bound electron (see appendix B.2 page79) as a single unit. This would halve the number of mode elements yet again and then require only one index identifier for each complete four part bound electron mode.

2.4 About the Hilbert Transform Commutator

Let us check, for simplicity in a single dimension, the introduction of the Hilbert transform on the time dimension alone. The Hilbert transform introduces a $\pi/2$ lag for positive frequencies:

$$\boldsymbol{H}_t\left(\text{cis}(\omega t)\right) \;=\; -1/\pi \int_{\mathbb{R}} \text{cis}(\omega(t+\tau))/\tau \; \mathrm{d}\tau \;=\; (1/t) \star \text{cis}(\omega t)/\pi \;=\; -\jmath\,\text{sign}(\omega).\text{cis}(\omega t)$$

... where $\star$ denotes convolution.

We require the field solutions of our matter wave equation to give rise to charge and current densities that concur with observations of induced electromagnetic fields and fluorescence whilst requiring no steady state radiation, ultraviolet catastrophe or other up conversion effects. To have such a solution field meet these empirical necessities and also the first degree Planck relationship $m_e c^2 = \hbar\omega$ between electronic charge, energy and frequency demands the presence of a field interaction involving the Hilbert transform.

Given elements of complex scalar field with the form $\psi_i(t,\mathbf{x}) = \text{cis}(\omega_i t)\Psi_i(\mathbf{x})$ then Hilbert transform commutator sesquilinear products from the ith of these to the four partial derivatives of the jth are according to equation EHt (2.1). To solve the modal wave equation for a bound state four such elements of field are needed to compose each distinct spin based electron mode, and in the steady state two of these oscillate at equal oppositely signed frequencies and two stand at zero frequency.

To accommodate analytically the fluctuations of a stochastic or otherwise incoherent system it is generally necessary to average, and in the case here present that could be over time, and possibly also over space. However the presence of the Hilbert transform embedded in a commutation structure without any averaging produces a smoothing effect instantaneously in some of the fluctuation terms, leaving only the low frequency and static terms present in the overall sesquilinear commutator product. It is thus realistic.

One might question why such manifestation of stochastic smoothing process can come about (and hence the Hilbert transform be justified) without the need to incorporate an explicit averaging process. To account for this we invoke here an underlying stochastic process at high frequencies in which mixing of frequencies is constrained to produce down conversion only. This is by the operation of the second law of thermodynamics on a form of limited stochastic disequilibrium of an underlying wave field. It admits of no up-conversion because it would correspond to a decreasing entropy of that underlying field. That is the nature of an underlying substrate field that we wish to model, but which is beyond the scope of this document.

Thus we shall proceed with the use of the Hilbert transform with, for the time being, no further detail of how the underlying process obtains this ability to suppress up conversions. We shall apply it with supposedly very good accuracy as operating instantaneously (as averaged from a much higher frequency substrate system), bypassing the need for the averaging normally used in correlations, and in Parseval and Plancherel theorems. Stochastic fluctuations, as exist, we then expect to be confined to extreme high frequencies around the Planck frequency, and to be negligibly small at or below the much lower Compton frequency.

Thus we shall derive frequency domain spectral expressions for the mixed frequency charge and current densities in place of the original time dimension partial differentials of the 4-space:

$$\begin{aligned}
\psi_i^*(t,\mathbf{x}).\boldsymbol{H}_t(\partial_t\psi_j(t,\mathbf{x})) &= \text{cis}(-\omega_i t)\Psi_i^*(\mathbf{x}).(-\jmath\,\text{sign}(\omega_j)\partial_t\text{cis}(\omega_j t))\Psi_j(\mathbf{x}) \\
&= \text{sign}(\omega_j).\omega_j.\text{cis}((\omega_j-\omega_i)t).\Psi_i^*(\mathbf{x}).\Psi_j(\mathbf{x}) \\
\text{... and similarly} & \\
\boldsymbol{H}_t(\psi_i^*(t,\mathbf{x})).\partial_t\psi_j(t,\mathbf{x}) &= -\jmath\,\text{sign}(-\omega_i).\text{cis}(-\omega_i t)\Psi_i^*(\mathbf{x}).\partial_t\text{cis}(\omega_j t)\Psi_j(\mathbf{x}) \\
&= -\text{sign}(\omega_i).\omega_j.\text{cis}((\omega_j-\omega_i)t).\Psi_i^*(\mathbf{x}).\Psi_j(\mathbf{x})
\end{aligned}$$

Therefore:

$$\begin{aligned}
&\psi_i^*(t,\mathbf{x}).\boldsymbol{H}_t(\partial_t\psi_j(t,\mathbf{x})) - \boldsymbol{H}_t(\psi_i^*(t,\mathbf{x})).\partial_t\psi_j(t,\mathbf{x}) \\
&\qquad = (\text{sign}(\omega_i)+\text{sign}(\omega_j)).\omega_j.\text{cis}((\omega_j-\omega_i)t).\Psi_i^*(\mathbf{x}).\Psi_j(\mathbf{x}) \qquad (2.4)
\end{aligned}$$

The Remarkable Smoothness

We see that an operator with the Hilbert transform commutator form of this last expression produces zero interaction between complex cisoid terms with opposite signs of frequency. (Keep checking that physical spectra comprehensively concur with these signs.) Furthermore, terms with like sign of frequency produce only difference frequency terms in the sesquilinear product. Thus, although still in need of direct deduction from say the stochastic under-model outlined in appendix G or any other such unified reasoning, it is nevertheless apparent that there exists a form of sesquilinear operator expression capable of producing down conversion but no up conversion ... a remarkable smoothness of interactions!

This feature is compatible with the otherwise puzzling observed presence of fluorescent emission and absorption along with absence of ultraviolet catastrophe (up-conversion) in the interactions between electrons bound to a common atom. It is therefore a valuable candidate process for our model.

Disappointingly, these Hilbert transform expressions are non-local and therefore non-causal. So until justification for this as a valid approximation is brought from the stochastic under-model this apparently undesirable feature of the model will remain with us.

Nevertheless this Hilbert transform commutator balancing structure is essential to achieve smoothness. If omitted that smoothness cannot be reintroduced merely by the bi-cisoid nature of the matter field terms.

Thus to avoid up-conversion causing fluctuations in charge and current densities there is no expression simpler than one having commutator structure in the operation of the Hilbert transform on the conjugate self product of the wave, though some simplification is visible in that the Hilbert transform operation can commute with the partial differentiation. Because of the apparent specificity of this possibility for such an expression, our quest for an account of how substrate scattering can produce the requisite behaviour in the system is directed toward mechanisms that correspond to this kind of analytical result.

Without needing to repeat this analysis in the spatial dimensions we may expect the same smooth features to occur in the behaviour of the Hilbert transform operations leading to the vector of current density. This is true in spite of the spatial factors of the eigenfunctions having continuous Fourier transforms, on account of the survival of linearity of the integral under Fourier transformation according to the Plancherel theorem.

These formulæ are based upon the convention that squared modulus of wave amplitude (positive intensity) originates positive charge unless inverted by premultiplier $\varpi = -1$ or other explicit statement, and that charge continuity is maintained by the corresponding sign convention for current. Also positive frequency ω of a complex variable $\mathrm{cis}(\omega t) = \exp(\jmath\omega t)$ corresponds to the temporal sequence $1, \jmath, -1, -\jmath$ as time increases.

2.5 Electromagnetic Propagation

Maxwell and Jefimenko

Maxwell, in using an internally coupled second order partial differential equation system

to define wave propagation, implicates the electric and magnetic fields in a mutual causal loop. He introduced this feature by combining Ampere's law with the notion of charge displacement currents, and thereby completed a loop of apparently causal relationships between electric and magnetic fields. However in spite of the validity of the relationship thus generated, the propagative causal mechanism can as well be described in terms of constant speed wave motion. In that alternative form the electric and magnetic field terms can be formed according to O.D.Jefimenko [ref] as retarded integrals from the original charge and current distributions. Since charge/current continuity is itself a necessary system property the coupling through the curl relationships in the Maxwell formulation can be viewed as redundant, and for some purposes best excluded from the system definition.

As an interesting Popperian aside we can assert that, given the acceptance of the charge continuity rule and constant speed of electromagnetic propagation, we should not make the Maxwellian claim that there is any curl based coupling between electric and magnetic fields. This is because that relationship will be there anyway, so there could be no way of ever proving the contrary. No such anomaly of scientific principle arises if we instead use Jefimenko's delayed integral expression of causation from the charge / current field to the electromagnetic field.

So in seeking a causal structure we may omit this weaving together of magnetic and electric effects from the form of the propagation rules without change to the system so long as we recognise the universal charge continuity in the source of these wave fields. Indeed, if we set out to make a full evaluation of the effects of all elements of charge and its associated current then we must omit any of these kinds of interactions if we are not to count them in twice. On such a basis we might say that there are then no interactions between the electric and the magnetic fields except via the agency of matter ... the coupled parts of the Maxwell equations are not required. The propagation can then be relegated to mere use of zero d'Alembertian constraint independently on the form of each of the six antiderivative vector elements constituting the electric and magnetic fields.

A further rationalisation is obtained by avoiding the couplings of the Maxwell equations in that the entire system can then be expressed in vectors of commensurate grade. These would usually be the first grade (polar) vectors E and H (electric and magnetic stresses), instead of the mixed grade vectors E and B (polar vector electric stress and pseudo-vector magnetic induction) that are typical of the Maxwell equations and the related Faraday tensor expressions.

Where the originating charge and current fields are sufficiently distant the option to describe the electromagnetic fields purely in terms of propagating waves, complete with their relationships, remains valuable. Because these fields behave linearly in vacuum (and also effectively in many dielectrics) it is permissible to add together independent terms described in these different ways.

It is perhaps intuitive to expect that dependence upon the curl couplings expressed in the Maxwell equations would at least be an essential feature of the propagative mechanism of transition from Fresnel to Fraunhofer regions, but even that turns out not to be the case.

So the approach used here is compatible with that proposed by O.D.Jefimenko in which

he uses delayed integrals to express the processes of electromagnetic propagation. (Refer also Liénard-Wiechert formulæ.)

Jefimenko Retarded Integrals
Framed in terms of conventional electromagnetic field symbols for free space the retarded integrals of O.D.Jefimenko expressed as functions on the 4-space coordinates $\vec{r}, t$ appear as follows:

$$\begin{aligned} \vec{E}(\vec{r},t) &= \frac{1}{4\pi\epsilon_0}\int\left(\frac{\rho(\vec{r'},t_r)\,\vec{R}}{R^3}+\frac{\vec{R}}{R^2c_0}\frac{\partial\rho(\vec{r'},t_r)}{\partial t}-\frac{1}{Rc_0^2}\frac{\partial\vec{J}(\vec{r'},t_r)}{\partial t}\right)\mathrm{d}^3\vec{r'} \\ \vec{H}(\vec{r},t) &= \frac{1}{4\pi}\int\left(\frac{\vec{J}(\vec{r'},t_r)\times\vec{R}}{R^3}+\frac{1}{R^2c_0}\frac{\partial\vec{J}(\vec{r'},t_r)}{\partial t}\times\vec{R}\right)\mathrm{d}^3\vec{r'} \end{aligned} \tag{2.5}$$

... where $\vec{R}=\vec{r}-\vec{r'}$, $R=|\vec{R}|$ and $t_r = t - R/c_0$ (the retarded time).

We can alternatively re-express the magnetic field in terms of what we might call the magnetising vector, here denoted by $\vec{V}$, where $\vec{H} = \mathrm{curl}\vec{V}$. Taking the opportunity in free space also to re-express permittivity ϵ_0 in terms of impedance z_0 and speed c_0 we then have:

$$\begin{aligned} \vec{E}(\vec{r},t) &= \frac{z_0c_0}{4\pi}\int\left(\frac{\rho(\vec{r'},t_r)\,\vec{R}}{R^3}+\frac{\vec{R}}{R^2c_0}\frac{\partial\rho(\vec{r'},t_r)}{\partial t}-\frac{1}{Rc_0^2}\frac{\partial\vec{J}(\vec{r'},t_r)}{\partial t}\right)\mathrm{d}^3\vec{r'} \\ \vec{V}(\vec{r},t) &= \frac{1}{4\pi}\int\left(\frac{\vec{J}(\vec{r'},t_r)}{R}+\frac{1}{c_0}\frac{\partial\vec{J}(\vec{r'},t_r)}{\partial t}\right)\mathrm{d}^3\vec{r'} \end{aligned} \tag{2.6}$$

When we use this Jefimenko derived form it simplifies things to better express in wave terms the coupling process with other matter upon which the electromagnetic field waves then impinge.

We may use a regular set of six antiderivative variables related to $\vec{E}$ and $\vec{V}$ to build a tensor mediating a matter wave equation such as will produce accelerative tendencies. The resulting kinematics can then be interpreted in terms of masses and the Lorentz forces by means of accounting for the accelerations via inverse application of Newton's second law of motion $\mathbf{f} = m.\mathbf{a}$.

Thus we have further simplified the model by avoiding any need for an independently defined basis of coupling to matter ... the wave solution process will itself bring about the requisite accelerative and wave vector effects through induction of phase twists and from that we can derive Newton's laws of motion.

Since in any full model of electrodynamics we must be concerned with the related rôles of matter and the electromagnetic field, there would appear to be some advantage in recognising these simplifications resulting from taking a simultaneous view of their joint behaviour.

Regarding the issue of relative permeability or permittivity, then for the microscopic case we may resort to the plain fact that all such effects that deviate from vacuum are the consequences of the presence of matter. By making due allowance for that matter present we may derive the macroscopic effects amounting to altered mean or distributed values of these spatial parameters.

Intra-atomic Near Field Propagation

The following should be tidied.

Return the following to "all grade 1 vectors". It is all we need.

From the charge/current real four-vector field $\mathbf{q}(\mathbf{x})$ on the physical Euclidean 3-space $\mathbf{x}$ the Maxwell system, or as we handle it here the Jefimenko system, generates the electromagnetic four-vector potential $\mathbf{a}(\mathbf{x})$. But that is only true if restricted to the static field or else the scalar potential must be allowed to be non-conservative. However for a host of generalizations of this model beyond the steady state, of which illumination by light from a far source is a common example, the inclusion of time to express these relationships on the physical 4-space is suitable thus:

$$\text{Jefimenko: } \mathbf{q}(t,\mathbf{x}) \to \mathbf{a}(t,\mathbf{x}) \quad \mathbf{q},\mathbf{a} \in \mathbb{R}^4; \quad t \in \mathbb{R}; \quad \mathbf{x} \in \mathbb{R}^3$$

Generation of the electromagnetic field follows in that it may be regarded as a gradient and a curl obtained from the elements of the four-vector potential field.

For atomic and small molecular structures the propagation delay of self-field perturbations is very small in relation to the periods of state evolution (i.e. in relation to the reciprocals of real parts of the characteristic poles of the matter-wave oscillatory system), so the time delay terms can usually be omitted for these cases and a simple prompt model can be used instead.

For the steady state static system these delays do not matter at all, and even with perturbations such as under luminous radiation around 10^{15}Hz the maximum delays determined by atomic diameter in this self-field propagation are still only around 10^{-17}sec and therefore usually negligible under these circumstances too.

> **Rabi Fluctuations**
>
> Note also that the atomic state perturbation frequencies that really concern us are in any case not those of the optical radiation itself for they have only a linear involvement. Rather it is the substantially lower frequencies of the Rabi gyrations of excitation states in the atom(s), so there is in fact even less of a problem with omission of that delay.

Thus it is practical and convenient to use the following prompt near field approximation (based upon normalised units) for the propagative processes of the electromagnetic system within any single atom, and even in rather larger structures such as substantial sized molecules. From the charge/current 4-vector $\mathbf{q}$ this approximation generates a 4-vector Helmholtz decomposition $\mathbf{a}$ of the static electromagnetic field under the Newtonian 3 dimensional potential kernel $1/(4\pi|\mathbf{x}|)$:

EMnear

$$\mathbf{a}(t,\mathbf{x}) \;=\; \frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\mathbf{q}(\xi)}{|\mathbf{x}-\xi|}\;\mathrm{d}\xi^3 \qquad (2.7)$$

This is the steady state near field part of the definition of the electromagnetic system in the retarded integral form as attributed to O.D.Jefimenko except that the result is expressed here as the four-vector potential instead of the usual electric and magnetic field vectors.

To set up the nuclear charges upon which to base simple electrodynamics models equation EMnear (2.7) can be evaluated with the requisite fixed concentrations of charge located at the desired nuclear centre(s) to be added to any fields produced by the electrons via the charge and current formulæ equation EHt (2.1) or Echarge (2.2).

The combination of the Matrix Klein-Gordon equation governing electron wave and the charge and current origination formulæ with Jefimenko (or the alternative Maxwell) equations for propagation of electromagnetic field is the complete definition of the electrodynamics model as used here. Our purpose is now to explore features and properties of its dynamic behaviour in the vicinity of atomic nuclei (but not inside those nuclei ... that remains for further descriptive extension of the model).

2.6 Interactions and Spectral Regimes

Einstein A&B Coefficients
Given a dipole strength of the wave function product between any pair of electron modes we may evaluate the Einstein A coefficient for spontaneous emission with an excitation in that pair. The corresponding B coefficient for interactive emission and absorption for that term then follows from the inherent dynamical Rabi process of interaction with an incident wave.

The General Structure of Couplings
An atom generally contains a superposition of a finite set of distinct electronic eigenmodal motions. If the atom is isolated there are circular periodic symmetries in the azimuthal plane of the electron mode waves. The strongest product terms giving rise to couplings are then dipoles that arise from mode pairs with a unit order difference between principal quantum numbers and having like intrinsic spin states. These produce the selection rules for possible couplings to the electromagnetic field.

When the circular symmetry of the electron field is spatially perturbed such as in molecules then other cases of such coupling can occur for mode pairs with no simple dipole product. Therefore the selection rules then become less distinct. See for example Cartmell & Fowles [CF1977] ... notes regarding absorption spectra in chapter 11 on p.213 and further discussion of d-d spectra in chapter 12.

Dynamics of Atomic State Transition Processes
For systems where the state undergoes a quantal change to a new attractor, such as through ionisation, then the resulting transient is a function of time that in most important cases is

likely to change relatively slowly compared to oscillations at the electron eigen frequencies near to the Compton frequency.

Where state changes occur too rapidly for this approximation, that is in extreme plasma-like cases, the Hilbert transform temporal integration loses its validity. (Check that. It is probably not true and the caution not necessary.) Then a more general model of the field must be constructed on a basis of the stochastic under model (see appendix G).

Most problems in electrodynamics do not extend to the latter case. Thus based upon limiting rates of change of atomic bound electronic state to around 1% of the quantal energy per Compton period these expressions are likely to be useful at practical temperatures (including up to say 10^6 °C, and higher for some purposes) and for resonant interactive absorption produced by radiation at intensities at least up to $10^5.f$ watt/m^2 at any frequency f up to X-rays (say up to 10^{19}Hz). This range is short of "extreme plasma", but it includes a broad "ordinary plasma" regime with outer electrons up to around 5KeV removed by ionization.

Atomic Emissions and Absorptions

These phenomena are the result of interaction between the electronic field of an atom and the local electromagnetic field. They involve absorptions and emissions, including fluorescence. The emissions can be either spontaneous or stimulated.

We can explore these various phenomena by studying the different kinds of multiplicative charge/current origination processes in equation Echarge (2.2). The picture so produced is markedly different from the conventional one based upon superposition of prior squared wave function amplitudes used in Schrödinger and Dirac formulations. It offers a more compactly coherent account of the details through the natural properties of sesquilinear arithmetic.

Expansion of Terms in Equation Echarge

The equation Echarge (2.2) is a summation of products of pairs of waves over a set in which there are four partial mode forms for each discrete electronic absolute frequency with a given intrinsic spin. This sum represents the density of scalar charge $q(t, \mathbf{x})$ and 3-vector current $\mathbf{i}(t, \mathbf{x})$, and may be regarded as a 4-vector $\mathbf{q}(t, \mathbf{x})$ of these field point values. The four parts, two of them at zero frequency, that constitute each electronic eigenmode are essential as the wave topological basis for intrinsic spin. No bound electron solutions exist without this four part intrinsic spin structure.

Thus for the n electronic modes that are bound to an atom we expect $4n$ of these partial mode waves constituting n compound eigenfunctions each comprising four strictly related parts. We index these partial mode waves by integers $0 < i, j \leq 4n$.

We shall study the processes for pairs within each such set of four electronic partial modes, and then also for pairs of the different kinds with each one taken from different electron modes, but still bound in a single atom.

Free electrons, where intrinsic spin does not play the same essential rôle, remain as a subject for separate study using the same charge/current origination expression in equation Echarge (2.2).

Self Interactions of Electronic Partial Modes
The self interactions deal with the cases where $i = j$. We can consider each of the four partial modes constituting any particular electronic mode.

The charge terms are multiplied by the frequency, so only the two dynamic (non-zero frequency) terms of the four contribute. The current terms depend upon a first spatial derivative so are generated only by the two azimuthally twisted terms of the four. There is no temporal fluctuation produced in either charge or current. The static flat partial mode contributes neither charge nor current effect ... it is only there to satisfy the balance of the d'Alembertian operator.

Cross Terms Within an Electronic Eigenmode
There are three cases of interest:
1 Both at zero freq. No charge is produced, but since one of the partial modes must be azimuthally twisted a magnetic moment is produced.

2 One at zero freq.

3 Neither at zero freq.

Cross Terms Between Electronic Eigenmodes
Further notes would be valuable here.

3 The Matter Wave Equation

3.1 Units and Variables

We now set out to describe both structurally and quantitatively the way in which the continuum charge in the electron fields of atoms, molecules and crystals becomes organized so as to manifest known physical phenomena and to behave in quantal units.

About the Units

Let us define unit charge origination as that occurring in a cube of unit edge containing a volumetric average unit intensity of the matter ψ-wave at unit temporal frequency. Looking at this in spherical polar co-ordinates we may define that when in isolation the potential at the surface of a sphere with radius r that is concentric with and contains such a (cube defined) unit charge is $1/r$. On this basis we may think of unit charge as radiating -4π lines of electric flux. Such an arrangement is typical of the c.g.s. system of units.

Alternatively to this we may make the charge definition using rectangular (Cartesian) co-ordinates with plane distributed charge. On this basis a plane having a uniform charge of one unit (as defined for a cube above) per unit area will have a unit change in slope of potential (with opposite sign to the charge) along any perpendicular path as it passes through the plane. This way the unit charge is defined as radiating a single line of electric flux.

The latter Cartesian arrangement is more in keeping with the SI system of units (though the 4π multiplier is still used in the basic definitions in SI too ... it has to appear somewhere), but for a simple equation in spherical co-ordinates the former, c.g.s style, is neater, and for that reason is used here. However the scaling is changed to $c_0 = z_0 = \omega_C = \hbar = 1$. These are respectively speed of light, impedance of space, electron Compton frequency, and Planck's constant, so that the free electron rest mass also comes out with unit value as $m_e.c_0^2 = \hbar.\omega_C$. It means that the calculation of static potential in free space at a spherical surface of radius r induced by the enclosed and sphero-symmetric charge $q(r)$ requires only division by r.

Regarding units of charge in the basic scheme used herein

If, all in basic (hypothetical) units, the Cartesian cuboid definition of charge unit has value $q_C = \sqrt{4\pi\hbar/z_0}$ and the basic spherical shell definition (unit thickness at unit radius) has value q_S then $q_S = 4\pi.q_C$. It must be remembered that the topology associated with charge quantum regulation introduces a change of scale, so further rescaling to the effective units will still then be necessary. The conventional quantal unit of charge is expressed as $\sqrt{4\pi\alpha.\hbar/z_0}$ where the unitless $\alpha = 1/137.0359998$ is the so called "Fine Structure Constant".

Quantum behaviour occurs in phenomena of both the original charge and the original magnetizing current kinds and they are both produced in the same type of wave field. The charge quantity is connected with the number of half wave ripples in a real wave (with

actual zero crossings) along a path starting at a positive nucleus and ending by tapering away at great distance, and the magnetic quantization is connected with the number of ripples (or rather, argument rotations of a complex variable having no zero crossings) along a path that forms a closed loop.

For an initial grasp we can concentrate upon the charge phenomena alone. This will allow us to develop a simple picture of the "orbitals" surrounding static structures of atomic nuclei. We shall then be in a position to augment this picture with the quasi dynamics of wave-orbital magnetic effects whilst still avoiding total generality. We may then take a view of how this can be generalized by various forms of perturbations, both static and dynamic, for yet further extensions of validity of the working approximations.

As a comment in passing, we might note that because of its topological symmetries the electron field of a charge neutral ground state single atom of a noble gas isolated in vacuo has no self generated magnetic fields at all, thus it offers an opportunity for simplified but realistic cases of study involving electric fields only.

3.2 The Modulated Klein-Gordon Equation

In order to bring out the regularity of structure based upon four dimensional tensors or matrices the governing equation for the matter wave system will first be developed in a Klein-Gordon format with its coefficients as a matrix that is a function of the space-time coordinates ... a modulated Klein-Gordon equation. It will then be developed and transformed from time to frequency to produce an equivalent matrix modulated Helmholtz equation. It will finally be expanded by using vector notation into a form which is in some ways easier to read but where the dimensional symmetries are less obvious.

The normalized four-vector differential operator in terms of equivalent conventional symbols is:

$$\Diamond \quad \equiv \quad [D, \nabla] \qquad \{ \equiv \quad \tau_C[\partial/\partial t,\ c_0.\partial/\partial x,\ c_0.\partial/\partial y,\ c_0.\partial/\partial z] \qquad \text{in practical units } \}$$

... where these are normally taken to be column vectors.

Using this symbolism the second order Minkowski differential operator (the d'Alembertian) can be written as $\Diamond^T \cdot \mathbf{M} \Diamond$, where the mid-dot serves to stress contraction by addition of elements following differentiation and the 4×4 matrix $\mathbf{M}$ contains the Minkowski metric in its diagonal thus:

$$\mathbf{M} \ = \ \begin{bmatrix} -1, & \mathbf{0} \\ \mathbf{0}, & \mathbf{I} \end{bmatrix}$$

When we wish to work in the frequency domain the complex wave variable $\psi(t, \mathbf{x})$ will be replaced for these static or relatively slowly varying systems by its Laplace transform $\Psi(s, \mathbf{x})$ where $s = \sigma + \jmath\omega$ is the generalized complex frequency. To write equations in this hybrid form the operator is a multiplier in the first, i.e. the frequency element, and three differentiators in the spatial elements:

$$\Diamond_s \quad \equiv \quad [s, \nabla] \qquad \{ \equiv \quad \tau_C[s,\ c_0.\partial/\partial x,\ c_0.\partial/\partial y,\ c_0.\partial/\partial z] \qquad \text{in practical units } \}$$

> ... where $s = \sigma + \jmath\omega$, and $s = \jmath\omega$ is a valid substitution for a steady oscillatory state, c_0 is the speed of light and $\tau_C = 1/\omega_C$ is the reciprocal of the electron angular Compton frequency.

The simplest relativistic wave equation with frequency dispersive velocity typical of the de Broglie matter waves has the Klein-Gordon form $[\Diamond^T \cdot \mathbf{M} \Diamond - 1]\, \psi(t, \mathbf{x}) = 0$.

For this equation to become part of our closed causal model requires that it shall be in some way modulated by the local state of the electromagnetic field. In this respect what we do here differs markedly from the Schrödinger and Dirac equations in that we take the distributed electromagnetic field to carry all interactions including the spatial modulation of self interaction arising locally from the form of the electron field. To do this we use the six distinct electromagnetic vector elements, three electric and three magnetic, to construct the 4×4 coefficient matrix, thereby producing a generalised form of the Klein-Gordon equation.

Relationship to Schrödinger and Dirac Equations

The conventional Schrödinger and Dirac equations do not address the issues at the core of the model described here. Because they are framed in the characteristic properties of a quantal entity (a particle), the values of energy and momentum treated by these conventional models are intensive. But in order to build the picture more fundamentally it is necessary to conceive of extensive variables so that the structures within such a particle can be studied in parts.

Here we form integrals over a continuum so as to arrive at the structure of a particle ... in the form of a soliton. Instead of considering just the entire energy or charge of a particle we work with the intensity in a wave field, and show how that can coalesce into a soliton with spatial integrals producing its characteristic values of charge, magnetic moment and energy. Furthermore we are free in the model described here to maintain a full view of the electronic matter fields and to take advantage of their linear properties in the short term under additive superposition.

In the conventional Schrödinger and Dirac equations the individual wave functions must be treated separately. Summation of their effects can only be performed after using some means of determining the details of each individual mode, and for this they are are conventionally treated as particles.

Here we start out with no kind of particulate Lagrangian description of electrons. As a result we are free to maintain a full view of the electronic matter fields in a superposition that has linear additive properties (at least in the very short term), and we follow through the consequences and opportunities of holding to that basis of field modeling.

In observing the above distinction it is then no surprise that contrasts are evident. First, the conventional Schrödinger equation does not produce a Lorentz invariant model, so our mission here is not comparable with that in its fundamentals. Regarding the Dirac equation, it maintains compliance with special relativity, but forces a separation of conjugate paired solutions in the pursuit of achieving first order differential structure. This is in order to describe individual particles as corresponding to

the separate eigenmodal solutions of the wave equation. Here we do not seek such discrete particulate entities, and even the notion of quantum is applicable at the level of atom, not of the electron.

Furthermore these conventional models and also the Bohm model have their structures based upon a scalar valued absolute potential field the idea of which appears to be inherited from a Lagrangian (and ultimately Newtonian) view of particulate motion that sees it as a necessity. But the electric potential field is clearly not in general a strictly conservative potential field. So here we instead use only local field attributes as the means for building structure, disallowing any single valued scalar potential as candidate for coefficients in any fundamental wave equation.

A result of this change is that the electron modal fields can be added when they are superposed (such as within an atom) in a common three dimensional physical space. Also this change of view allows us to handle the so called "gauge" variables differently in this model as will emerge here below.

Schrödinger and Dirac equations do not solve for waves that can literally represent electrons, but only the probabilities of finding electrons. By comparison we are here describing a system of spatio-temporal waves that represent, in their behaviour, the phenomena we observe in electronic mechanics. Processes of observation with their inherent uncertainty are then treated by a separate analysis, but derived from the same mechanics.

These waves that are thus posited as constituting electrons then tend to have frequencies that are very high, notably just below the Compton frequency for the bound state electrons, and above the Compton frequency for free electron states. By analysing the electron waves in the temporal frequency domain it is possible to extract a non-oscillatory factor ... a static wave profile to go along with the discrete eigenfrequency to define each mode.

Incorporation of Rabi Processes

To produce from the electron wave equation of the wave model an excitation state varying model (i.e. for interactions without ionic changes) it is necessary to study the Rabi fluctuations of excitation energy transfers between the electron modes and local electromagnetic waves. These occur at low frequencies, typically below optical frequencies, often in the radio frequencies, and for special systems like narrow band lasers and atomic clocks sometimes as low as 1Hz.

Fortunately these fluctuations typically introduce only small amounts of change in the mode shapes and frequencies. Thus by neglecting these small changes the wave model offers an attractively concise system describing approximately the fluctuation dynamics of electron modal energies, and these would typically, though not necessarily, be centred about their ground state Fermi distribution values.

The simplest equations representing these kinds of dynamical processes equate the first order time derivative of each modal (complex) excitation state to the sum of a number of interaction terms between the electromagnetic field and pairs of electron modal excitations.

Thus we begin with a matrix modulated Klein-Gordon equation: **EKG**

$$[\Diamond^T \cdot \mathbf{k}(t,\mathbf{x})\Diamond - 1]\ \psi_n(t,\mathbf{x}) \ = \ 0 \tag{3.1}$$

... where the matrix $\mathbf{k} = \mathbf{M} + \mathbf{f} + \mathbf{w}$ comprises:

- $\mathbf{M}$ the Minkowski metric $(-1, 1, 1, 1)$ in its diagonal
- $\mathbf{f}$ an electromagnetic field tensor placed in the off-diagonal elements
- $\mathbf{w}$ a perturbation added to the diagonal.

(Note ... See the development below:
$\mathbf{f}$ is not the Maxwell field tensor commonly used in physics.)

Then the description of the full wave function is: **EKGS**

$$\psi(t,\mathbf{x}) \ = \ \sum_n \mathrm{cis}(\omega_n t).\Psi_n(t,\mathbf{x}) \tag{3.2}$$

... where $\Psi_n(t,\mathbf{x})$ is the constant or at most slowly fluctuating amplitude factor in the nth modal wave function with frequency ω_n.

Here $\mathbf{k}$ and hence also Ψ_n are still treated as functions of time. This kind of model is of use where the frequencies of fluctuations or perturbations of these variables is low compared to the oscillation frequency ω_n, and that covers nearly all of the cases we encounter. However that expression of a dependence upon time will be omitted altogether for conciseness where we wish to establish only the steady oscillatory state of the electron system.

Nonlinear Electron Excitation Processes

This dependence of coefficients upon time is the basis of modeling interactions between incident electromagnetic field and atomic electrons. Electron excitation fluctuations called "Rabi processes" give rise to bilinear interactions for weak incident waves producing spectral absorption and emission phenomena, and nonlinear processes can arise when there are sufficiently large Rabi gyrations in the state space of electron excitations that is referred to as the "Bloch sphere" [AE1975].

These nonlinear phenomena occur for relatively strong and/or coherent electromagnetic fields. For a field to be "strong" in this sense requires its energy density $\rho > \hbar.\Delta\omega/v$ where v is the electronic modal interaction volume and $\Delta\omega$ is the bandwidth of the radiation. Allowance must also be made for the decohering effects of any noise field present.

However these Rabi fluctuations, whether strong or weak, are typically slow by several decimal orders compared with the resonant frequencies of the electrons around the Compton frequency, and so they can be treated as low frequency perturbations about the steady state. Thus a steady state analysis remains a powerful basis for approximations.

Freedom from Electron Frequency Up-conversion

In the first order of interactions there is no spontaneous rise at high frequencies of any direct intermodulation between electromagnetic and matter wave fields (neither "UV catastrophe" nor energy loss by radiation from steady state bound electrons). There is only a generally weak background coupling induced between the ψ-wave modes by the thermal and zero point noise of the quiescent electromagnetic field. This is an important and simplifying feature, but details of this aspect of this wave model will not be further developed here.

3.3 Coupling Coefficients

We shall now set about determining how the structure of the electromagnetic field can be used to determine the elements of the coupling coefficient array $\mathbf{k}(\mathbf{x})$ for each point in space.

Differential Orders in Couplings

Our expressed system of physics is self referential in that we and all of our apparatus of observation, measurement and expression are part of that same system. Any expression of a fundamental relationship is subject to this limitation. So the use of such relationships as parameters for a system purporting to originate beneath and to subsume the processes of observation may need reinterpretation to exclude the inherent effects of the measurement process. The necessity arises here in respect of the coupling coefficients from the electromagnetic field.

We know that all observations rely upon original discrete state changes in which charge quanta are of invariant magnitude. These discrete changes are related to the process oscillation frequencies involved always such that their effect in terms of energy is proportional to oscillation frequencies (Planck's constant $\hbar$ always relates $E = \hbar\omega$). Because of this we must adjust what we mean by a coupling coefficient when applying it below the level of perception to remove this bias proportional to frequency.

We wish to retain the usual way of writing differential equations, but we may note that the coefficients only ever enter our equations surrounded by differentiation operators to produce a homogeneous second order structure. Thus we may compensate the unwanted frequency proportionality by applying an abstract integration to the respective coefficients derived from these observable couplings. This leads to what might be called indefinite integrals, or more characteristically, antiderivatives as entities we wish to use as coefficients. The abstract integration introduced in this way is absorbed by the associated differentiation operations so that no arbitrary constants of integration need appear in the solutions of the equations so formed.

A similarly conceived abstract integration, in this case an anticurl, is required in the spatial domain (essentially for magnetic effects) as we have argued for the temporal domain (essentially for electric effects).

Such introduction of abstract antiderivatives may be seen as the price to pay for establishing a homogeneous second derivative matrix operator. The off diagonal elements contribute only in first derivative order (the remaining defferentiation being associated with the wave variable), so we need to off-set this excess differentiation with an antiderivative operation upon all of these off diagonal matrix elements.

This approach to modeling reduces the reliance upon using the abstraction of gauge theories that is necessary in all ensuing work if such an abstract compensation is not introduced at the fundamental level. Though perhaps abstruse at the point of introduction here, in subsequent application work this helps improve the intuitive clarity.

Use of Antiderivative Coefficient Elements in the Matrix

The conventional practice of using a four-vector of potentials goes along with an asymmetrical vector grade structure of wave system causality and will not be used here. (That also means that we do not adhere to the canonical formalism of quantum mechanics.) Instead the use of an antiderivative in the magnetising vector $\mathbf{v}$ (it is the anticurl of $\mathbf{v}$ and we denote it by $\mathbf{u}$ here) will be matched by another antiderivative (it is temporal, and denoted by $\mathbf{g} = -\int \mathbf{e}\,\mathrm{d}\mathbf{t}$ here) associated with the electric field vector $\mathbf{e}$.

Thus (along with closure of the causal loop structure for matter) we arrive at a pair of 3-vectors with uniform first grades instead of the conventional single 4-vector with an inconsistent grade formulation.

So we use symbols:

Electric 3-vector field strength	$\mathbf{e} = (e_1, e_2, e_3) = -\mathrm{d}\mathbf{g}/\mathrm{d}t$
Electric 3-vector antiderivative	$\mathbf{g} = (g_1, g_2, g_3) = -\int \mathbf{e}\,\mathrm{d}t$
Magnetic 3-vector field strength	$\mathbf{h} = (h_1, h_2, h_3) = \nabla\times\mathbf{v} = \mathrm{curl}\,\mathbf{v}$
Magnetising 3-vector	$\mathbf{v} = (v_1, v_2, v_3) = \mathrm{curl}^{-1}\,\mathbf{h} + \mathrm{const} = \mathrm{curl}\,\mathbf{u}$
Magnetising 3-vector anticurl	$\mathbf{u} = (u_1, u_2, u_3) = \mathrm{curl}^{-1}\,\mathbf{v}$

The usual electromagnetic field tensor used in physics is built from mixed grade-1 (polar vector) and grade-2 (axial or pseudo-vector) derivative elements, and as such is suited to the description of the overall electromagnetic system as a closed causal system (through incorporation of Ampere's Law). Here we instead regard the electromagnetic field as causal only in its rôle in providing coefficients in the matter wave equation, not as a complete system in itself. (See the related preference for the Jefimenko approach in section 2.5 above.)

So we arrange the causal influence to arise with similar grade structure from all of the elements of the coefficient matrix we use in the wave equation, and to that end we make these all grade-1 types of elements. We achieve this by expressing in terms of two vectors of three dimensions each $\mathbf{g} = -\int \mathbf{e}\,\mathrm{d}t$ and $\mathbf{u} = \mathrm{curl}^{-1}\,\mathbf{v}$, being respectively a temporal antiderivative of the grade-1 electric field 3-vector $\mathbf{e}$ and a spatial anticurl of the grade-1 magnetising field 3-vector $\mathbf{v}$. This leaves these two coefficient vectors indefinite regarding their respective constants of integration, but when combined with the differential

structure of the wave equation these indefinitions disappear, as will be shown below. The antiderivative nature of these coefficients may then be seen as no more than a convenient means to unify the matrix expression of the required second order differential operator in the matter wave equation.

For an elemental analysis we only need vacuum relationships since it is always matter that will give rise to any alterations of effective permeability and permittivity of space.

The completion as a description of the causal process that happens to maintain a relationship between these pairs of electromagnetic field vectors will remain in the underlying substrate field correlations, and that will be developed elsewhere. Here we accept it just as a fact, according to the form of the Jefimenko expressions.

Unitarity of the Differential Operator

The matrix in the differential operator contains all the possible coefficients that can appear in the linear partial differential equation without disturbing its differential homogeneity. Note that it is not the rather commonplace matrix referred to variously such as the "Maxwell Field Tensor" or the "Faraday tensor" ... this one is about interactions with the electron wave field and is constructed differently.

All interactions that maintain the structure of each electronic modal wave function are mediated by our coefficient matrix. Therefore we must take care to see that it is unitary everywhere in order that each electron wave eigenmode can display constant eigenvalue (coherent frequency) at all points in the space it occupies (see also appendix E.1).

The process maintaining this can be analysed at a deeper and unifying stochastic level of the model where it maintains conservation of a stochastic correlative intensity (a field autocorrelation). The description of that lower layer is not yet published (as of 16 March 2014) except for mention here in appendix G, but such a unitarity is anyway intuitively plausible. That description will be referred to in future versions of this essay when it is available.

Consistent with these rules the elements of the electromagnetic field are then built into the matrix.

Perturbed Diagonal of the Coefficient Matrix

For our coefficient matrix to remain unitary in the steady state oscillatory operation of the wave function there are constraints that must then apply to the row-wise and column-wise norms of the matrix elements. For clarification of these ideas the reader is referred here also to "The Spectral Theorem" in mathematics. This produces small perturbations of the otherwise unit magnitudes of the diagonal elements of the coefficient matrix. (See also appendix E.)

For cases where the matrix diagonal perturbations are small enough some analyses can omit them. So for this first analysis with electrodynamics as its chief concern these perturbations of the diagonal elements of the coefficient matrix will mostly be omitted. Even so, concern for the effects upon the leading element of the matrix due to the electric part of the field from the atomic nucleus may need to be dealt with,

and there are other issues too that may be important for some purposes (see also section 7.11 regarding Aharonov-Bohm effects).

We wish to accommodate the electric and magnetic fields in ways that suggest the requisite matter wave group accelerative behaviour that is typical of physics (Lorentz force effects etc.). Also by keeping the leading row and column of the coefficient array as containing the electric field then divergence of the electric field, corresponding to charge density, enters a process of spontaneous quantization of the charge in the solutions of the equation.

To maintain the uniform phase steady oscillatory state of a matter wave we require that the differential operator matrix shall be unitary over all of $\mathbf{x}$. The diagonal perturbations so introduced are not in any simple way related to the four-vector potentials, but are determined by constraints as described in appendix E.1, involving and effectively determining the local structure of the electromagnetic field. To accommodate these perturbations let us add a diagonal matrix $\mathbf{w} = (w_{00}, w_{11}, w_{22}, w_{33})$ comprising a scalar $\mathbf{w}_e = w_{00}$ and a 3×3 diagonal sub-matrix $\mathbf{w}_m = (w_{11}, w_{22}, w_{33})$.

So the coupling coefficient array is then:

$$\mathbf{k}(\mathbf{x}) \;=\; \mathbf{M} + \mathbf{f} + \mathbf{w} \;=\; \begin{bmatrix} -1+w_{00} & -g_1 & -g_2 & -g_3 \\ g_1 & 1+w_{11} & \jmath u_3 & -\jmath u_2 \\ g_2 & -\jmath u_3 & 1+w_{22} & \jmath u_1 \\ g_3 & \jmath u_2 & -\jmath u_1 & 1+w_{33} \end{bmatrix} \tag{3.3}$$

... where $\mathbf{M}$ is the Minkowski diagonal matrix, $\mathbf{f}$ is the off-diagonal antiderivative electromagnetic field matrix and $\mathbf{w}$ is the diagonal perturbation.

The diagonal perturbation $\mathbf{w}$ is in many cases (e.g. in the lighter atoms) very small compared to the unit magnitudes of the Minkowski diagonal matrix elements. So for an initial analysis here below it will only be indicated as an undefined added term, ready for later evaluation in detail if and when needed.

Signs of the Off-diagonal Matrix Elements
The signs of the off-diagonal elements in the matrix are antisymmetrical. Here in equation (3.3) the electromagnetic field terms constituting this coefficient array are negated overall to accommodate the negative sign convention for electronic charge, but below at equation (3.8) the sign of charge will be separated into the symbol ϖ "varpi".

Real or Imaginary Off-diagonal Matrix Elements
The use of antiderivatives in the off-diagonal elements in (3.3) combined with the hybrid format of Fourier transform to the time dimension but not to the space dimensions means that the electric off-diagonal elements of the matrix shall be real for the electric terms and imaginary for the magnetic terms. With no Fourier transform present the off-diagonal elements would all be imaginary.

The Use of Temporal Fourier Transforms

Beyond producing a consistent causal wave model this way, a particular separation by different treatment of the time dimension simplifies our path to analysis of the steady or near-steady states of atomic systems. The main effect of this lies in the way we shall usually resort to a transformation into the frequency domain for the time dimension, but not in the three space dimensions ... as will appear in the text. The motive for this asymmetrical treatment of analysis can be grasped from the fact that persistence of matter-as-waves exists as steady oscillation as a function of time, but not of the spatial dimensions.

Because of issues such as these it is desirable to work in the hybrid frequency domain, transforming time into frequency, thereby taking advantage of the natural separability of the modal solutions according to their respective frequencies. Such an approach is possible so long as we are analysing cases sufficiently near to the steady oscillatory state of the electron matter wave fields, allowing only limited static and dynamic perturbations from a given steady oscillatory state of electrons.

3.4 Frequency Domain - Modulated Helmholtz Equation

Starting from the Klein-Gordon equation EKG (3.1) we may derive an equivalent Helmholtz equation with the advantage of a more natural way of analysing in terms of separate electronic modes with distinct temporal frequencies.

We shall simplify the expressions by omitting the diagonal perturbations of the differential operator matrix. This is acceptable because they are very small for the practical electrodynamics in which we are interested, and they can be subsequently reintroduced as perturbations upon the final form of the equations if and when required. (See for instance equation (3.7) EMHM below.)

Substituting the matrix from (3.3) into equation (3.1) and expanding the lozenge symbols:

$$\left[[\partial_0, \nabla]^T . \begin{bmatrix} -1 & -g_1 & -g_2 & -g_3 \\ g_1 & 1 & \jmath u_3 & -\jmath u_2 \\ g_2 & -\jmath u_3 & 1 & \jmath u_1 \\ g_3 & \jmath u_2 & -\jmath u_1 & 1 \end{bmatrix} [\partial_0, \nabla] - 1 \right] \psi_n(t, \mathbf{x}) \; = \; 0 \tag{3.4}$$

We need to separate some differential product rule terms. Note that mixed derivative terms that do not differentiate the matrix coefficient elements cancel out due to the anti-symmetric disposition of the off-diagonal elements. So setting bracket precedences to allow differentiation to operate via] but not via) the full operator expansion is:

$$\left[\begin{bmatrix} -\partial_{00} & -(\partial_0 g_1)\partial_1 & -(\partial_0 g_2)\partial_2 & -(\partial_0 g_3)\partial_3 \\ (\partial_1 g_1)\partial_0 & \partial_{11} & \jmath(\partial_1 u_3)\partial_2 & -\jmath(\partial_1 u_2)\partial_3 \\ (\partial_2 g_2)\partial_0 & -\jmath(\partial_2 u_3)\partial_1 & \partial_{22} & \jmath(\partial_2 u_1)\partial_3 \\ (\partial_3 g_3)\partial_0 & \jmath(\partial_3 u_2)\partial_1 & -\jmath(\partial_3 u_1)\partial_2 & \partial_{33} \end{bmatrix} - 1 \right] \psi_n(t, \mathbf{x}) \; = \; 0 \tag{3.5}$$

We are now in a position to replace the Klein-Gordon format with a Helmholtz equation by transforming the time dimension into the frequency domain. Suitable to the steady state atom we restrict the generalised frequency s to a discrete (eigen)value $s_n = \jmath\omega_n$ for the

nth modal term in the electronic wave. The dispersion constant -1 is swapped with the matrix leading element ω_n^2 to leave a matrix that is independent of this mode frequency:

$$\left[\left[\begin{array}{cccc} -1 & -E_1 \star \partial_1 & -E_2 \star \partial_2 & -E_3 \star \partial_3 \\ (\partial_1 E_1)\star & \partial_{11} & \jmath(\partial_1 U_3) \star \partial_2 & -\jmath(\partial_1 U_2) \star \partial_3 \\ (\partial_2 E_2)\star & -\jmath(\partial_2 U_3) \star \partial_1 & \partial_{22} & \jmath(\partial_2 U_1) \star \partial_3 \\ (\partial_3 E_3)\star & \jmath(\partial_3 U_2) \star \partial_1 & -\jmath(\partial_3 U_1) \star \partial_2 & \partial_{33} \end{array} \right] + \omega_n^2 \right] \Psi_n(s, \mathbf{x}) \; = \; 0 \qquad (3.6)$$

Here the multiplications of matrix elements with the spatially differentiated variable ψ_n have been transformed in time to convolutions (using the symbol $\star$ for hybrid convolution in frequency only). Also the antiderivative form of each electric coefficient has been replaced with the electric field value by cancelling with its associated temporal differentiation. (They commute under the Laplace transform.)

This hybrid frequency domain matrix form of the equation in (3.6), including a term $\mathbf{W}$ reintroduced under temporal Laplace transformation from equation (3.3) to allow for the small diagonal perturbation (omitted above for brevity), can be compacted into the following notation:

EMHM

$$\left[\left(\diamondsuit_1^T \cdot \mathbf{K}(s, \mathbf{x}) \right) \star \diamondsuit_1 + \omega_n^2 \right] \Psi_n(s, \mathbf{x}) \; = \; 0 \qquad (3.7)$$

... where:

$$\mathbf{K}(s, \mathbf{x}) \; = \; \mathbf{M}\delta_\omega + \mathbf{F} + \mathbf{W} \; = \; \left[\begin{array}{cccc} W_{00} - \delta_\omega & -E_1 & -E_2 & -E_3 \\ E_1 & W_{11} + \delta_\omega & \jmath U_3 & -\jmath U_2 \\ E_2 & -\jmath U_3 & W_{22} + \delta_\omega & \jmath U_1 \\ E_3 & \jmath U_2 & -\jmath U_1 & W_{33} + \delta_\omega \end{array} \right] (s, \mathbf{x})$$

This rather comprehensive expression allows the matrix in its temporal Laplace transform form to convey interactions entering dynamically via any of the coefficient array elements. It offers means for handling linear dynamic perturbations (in generalised frequency domain over s) through such as light irradiation of the electron field. Where no such fluctuation of coefficients occurs they can be left individually untransformed and the corresponding convolutions left as multiplications. In particular, fluctuations of the diagonal elements are rarely studied (consider such as para/diamagnetism and issues of electronic stability) so can usually be disregarded, as they will be below.

The δ_ω symbols represent Dirac impulse functions (with zero offset) over the temporal frequency dimension. They are introduced here as Laplace transforms of the unit diagonal elements to allow uniform operation of the matrix $\mathbf{K}$ with the hybrid frequency convolution (denoted by $\star$) to its right. Apart from uniformity of notation, such detail is only required for dynamic perturbations of the state of the diagonal.

We shall here confine our attention to the simpler cases with constant values to the the diagonal elements. On that basis we may collect the terms from the simpler matrix in equation (3.6) to reduce the differential operator to a vector expression. Extracting $(\operatorname{div} \mathbf{E}) \star$ from the first column and $\mathbf{E} \star \operatorname{grad}$ from the leading row:

$$\left[\left[\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & \partial_{11} & \jmath(\partial_1 U_3)\star\partial_2 & -\jmath(\partial_1 U_2)\star\partial_3 \\ 0 & -\jmath(\partial_2 U_3)\star\partial_1 & \partial_{22} & \jmath(\partial_2 U_1)\star\partial_3 \\ 0 & \jmath(\partial_3 U_2)\star\partial_1 & -\jmath(\partial_3 U_1)\star\partial_2 & \partial_{33} \end{array} \right] - \mathbf{E}^T\star\nabla + (\nabla^T\mathbf{E})\star + \omega_n^2 \right]$$

Clearing leading row and column and extracting diagonal:

$$\left[\left[\begin{array}{ccc} 0 & \jmath(\partial_1 U_3)\star\partial_2 & -\jmath(\partial_1 U_2)\star\partial_3 \\ -\jmath(\partial_2 U_3)\star\partial_1 & 0 & \jmath(\partial_2 U_1)\star\partial_3 \\ \jmath(\partial_3 U_2)\star\partial_1 & -\jmath(\partial_3 U_1)\star\partial_2 & 0 \end{array} \right] + \nabla^2 - \mathbf{E}^T\star\nabla + (\nabla^T\mathbf{E})\star + \omega_n^2 - 1 \right]$$

The remaining matrix contains curl $\mathbf{U}$ which is the magnetising 3-vector field $\mathbf{V}$, and it is followed by a differentiator leaving the equation as follows. The above derivations were all related to electron fields, but we may generalise to charge of either sign by re-introduction of the symbol "varpi" as $\varpi = +1$ for positive charge and $\varpi = -1$ for negative where needed.

Emodedynamic

$$\left[\nabla^2 + \varpi(\mathbf{E}(s,\mathbf{x}) - \jmath\mathbf{V}(s,\mathbf{x}))^T\star\nabla - \varpi(\nabla^T\cdot\mathbf{E}(s,\mathbf{x}))\star + \omega_n^2 - 1 \right] \Psi_n(s,\mathbf{x}) \;=\; 0 \qquad (3.8)$$

For the unperturbed steady state where the coefficient array is static this equation may be simplified by removing the Laplace transforms from the coefficients and replacing the convolutions by multiplications.

Emode

$$\left[\nabla^2 + \varpi(\mathbf{e}(\mathbf{x}) - \jmath\mathbf{v}(\mathbf{x}))^T\cdot\nabla - \varpi(\nabla^T\cdot\mathbf{e}(\mathbf{x})) + \omega_n^2 - 1 \right] \Psi_n(\mathbf{x}) \;=\; 0 \qquad (3.9)$$

We may scale the fundamental constants to SI units as:

$c_0 = 1/\sqrt{\mu_0\epsilon_0}$	Light speed of free space	2.99792×10^8	metre.sec^{-1}
$z_0 = \sqrt{\mu_0/\epsilon_0}$	Impedance of free space	376.730	volt.amp^{-1}
$\omega_C = 1/\tau_C$	Compton angular frequency	7.76344×10^{20}	sec^{-1}
$\hbar = m_e c^2\tau_C$	Planck angular frequency constant	6.62618×10^{-34}	volt.amp.sec^2

The natural unit of electric potential expressed in SI is then $\sqrt{4\pi.\hbar.z_0}.\omega_C$ volt and the electron rest mass is related by $m_e.c_0^2 = \hbar.\omega_C$ joule. Notice that changing from the spherical style of relationship between electric field divergence and charge density to the Cartesian style of SI units introduces the factor 4π here. It must also be remembered that the topology for charge quantum regulation produced by equation (3.9) involves a self reference that affects measurement, so further rescaling will be necessary in any final computation of effective fine structure constant.

Putting these unit scaling factors into (3.9) as metre^{-2} throughout gives:

Emodeunits

$$\left[\nabla^2 + \varpi\frac{(\mathbf{e}(s,\mathbf{x}) - \jmath(\omega_C z_0/c_0)\mathbf{v}(s,\mathbf{x}))^T\cdot\nabla - (\nabla^T\cdot\mathbf{e}(s,\mathbf{x}))}{\sqrt{4\pi.\hbar.z_0}.\omega_C} + \frac{\omega_n^2 - \omega_C^2}{c_0^2} \right] \Psi_n(s,\mathbf{x}) \;=\; 0 \qquad (3.10)$$

Differentials of Matrix Coefficients

It is important to recognize that the expansion of equation (3.4) produces two terms in equation (3.5) through the differentiation of a product by the left-hand differentiator. It is the product of the matrix $\mathbf{k}(\mathbf{x})$ and the right-hand differential that has to be treated to the Leibniz rule for differentiation of a product. This differentiation occurs because the strict local causality of this model requires it always to accommodate the detailed spatial modulation of the electromagnetic field. It is this self interaction of the electron field that leads to spontaneous quantization.

The anti-symmetry of the off-diagonal elements causes the right-hand term of the product differential to disappear for all but the diagonal elements of the matrix. However, where the left-hand differentiator differentiates the matrix it produces the vital terms that give detailed structure to the system. In particular it produces a term in which the divergence of $\mathbf{e}$ (proportional to charge density) modulates the differential operator as a function of the 3-space co-ordinates. That completes a cubic system defining the structures of resonant matter wave solutions. Within each independent region of these the overall atomic charge can only take quantal values determined by the stochastic equilibrium of the underlying cosmological substrate agitation process.

Apart from the simplifying omission of matrix diagonal perturbations (see appendix E) this is a quite general steady state equation incorporating couplings both from afar and arising from the local atomic wave components. It will now be used to show in a simple case how it can give rise to a process culminating in spontaneous quantization of overall charge, and that can then be generalized across atomic, molecular and crystalline structures.

Here the wave function is expressed on the 4-space $(s, \mathbf{x})$, and for the steady state can only comprise discrete terms in the frequency domain. To keep this option present the notation used here will be to include the frequency domain variable s or equivalently $\jmath\omega$ present in the function argument only when the high frequency oscillatory factor is included within.

The overall wave function can then be assembled from the individual discrete frequency based terms thus: **EMHS**

$$\Psi(s,\mathbf{x}) \;=\; \sum_n \Psi_n(s,\mathbf{x}) \;=\; \sum_n \delta(s-\jmath\omega_n).\Psi_n(\mathbf{x})\;. \qquad s=\jmath\omega \tag{3.11}$$

Some Matrix Inhomogeneities

These 4×4 coefficient matrices, in the various forms depicted above, are peculiar in the degree of inhomogeneity in the entities forming the elements. In the Helmholtz equation form of (3.7) and (3.4) the leading row and column have off diagonal elements that are field strengths which decay inversely with square of distance whereas in the remainder of the matrix the off diagonals are the anticurl of the magnetising field, and that decays inversely with distance.

This arises because of the substantial asymmetry between the geometric nature of the 1-time and 3-space dimensions. Note particularly that the magnetic terms appear disposed such as to induce a curl effect, and hence they too give rise to inverse square decay in that

respect, whereas in the single dimension of time no such disposition of elements is either possible or necessary. As a result when equations (3.7) and (3.4) are combined to produce the electron mode equation Emode (3.9) this apparent irregularity in the nature of the coefficients disappears.

Electron Field Equilibria
Externally applied electromagnetic fields cannot generally be balanced to a null by adaptation of the form of the electron field. The extent to which these constraints can be accommodated in equilibrated structure of the electromagnetic field is only enough to balance the induction terms coming from local charges and currents. Far field terms come from external sources beyond the region of direct effect from the locally coherent matter wave. By way of analysis their potentials fortunately are often expressible by just their mean and gradient.

These unbalanced external fields induce dynamic phase twist of the matter wave solution. As a consequence of this they give rise to translational accelerative (the origin of Lorentz forces) and rotative motion in the group form of that wave field and also to magnetic Aharonov-Bohm phase effects.

Though derived and expressed this way for the static wave group case the behaviour is still invariant under Lorentz transformation because throughout the system the space-time differentials are all homogeneously of second order.

We now look for solutions of equation EMHM (3.7) for each possible electron wave mode Ψ_n at frequency ω_n. For simplicity in a first electron wave function development we allow the possibility of leaving out certain quadratic field strength effects by omitting the perturbations of the matrix diagonal (see section 7.11). (This might yet turn out to be unacceptable at least regarding the electric field produced by the nuclear charge.)

4 Identifying the Quantal Ripple

4.1 Spheroidal Mode Equation

First let us state, for reference, the form of the Laplacian in three dimensions with spherical coordinates ... radius r, colatitude ϕ, azimuth θ:

$$\nabla^2 f = \frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\left(\frac{\partial}{\partial \phi} + \cot(\phi)\right)\frac{\partial f}{\partial \phi} + \frac{\csc^2(\phi)}{r^2}\frac{\partial^2 f}{\partial \theta^2} \tag{4.1}$$

$$\text{or equivalently} \qquad r^2\nabla^2 f = \frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{\sin(\phi)}\frac{\partial}{\partial \phi}\left(\sin(\phi)\frac{\partial f}{\partial \phi}\right) + \frac{1}{\sin^2(\phi)}\frac{\partial^2 f}{\partial \theta^2}$$

The following equation (4.2) is a formulation in atomic centred spherical coordinates of the electron mode equation as derived from its spinless Cartesian form in equation Emode (3.9). Although working in three dimensions, we need only the two coordinates radius r and colatitude ϕ here because we are modeling the isolated atom, and that always has the form of circular periodic symmetries in azimuth θ. These symmetries produce circularly uniform densities of electronic charge and magnetizing vector. Equation (4.2) also incorporates the effect of the nth modal magnetic quantum number m_{ln} (orbital spin).

It is necessary to include orbital spin index m_{ln} and also the frequency effect of the intrinsic spin factor in this modal residual equation (4.2) because they are independently determined for each mode whereas the spatial part of the intrinsic spin factor has a similar form in all modes and is in any case needed in the radial ripple H function to determine its convergences in approach to origin, to axis generally and to infinite periphery. Therefore the intrinsic spin factor spatial gradient is carried separately in the bound state ripple equation (5.6) as shown below. Although split in this way the effects of the separated intrinsic spin terms in the respective differential operators recombine in the product $G_n H$ (see appendix B for a fuller outline of the intrinsic spin factor wave function).

The four parts of the intrinsic spin factor are of equal amplitude with frequencies respectively $\omega = +\omega_n, 0, 0, -\omega_n$. Thus to arrive at the overall effect of the four part spin factor for the bound electron state it is necessary to halve the squared modal frequency term along with the unit constant determining the Compton frequency. (Check this and justify algebraically why the halving factor has to apply to the unit constant as well as to the ω_n term. Note that the Compton frequency physics would be crazy if it did not!)

So the two dimensional expression in $\mathbf{r} = \{r, \phi\}$ of the residual equation for an electron in the spheroidal three dimensional atom with charge and current densities circularly symmetrical in azimuth and with azimuthal terms in both orbital and intrinsic spins present is:

$$\left[\ \nabla^2 + \varpi(\jmath\mathbf{v}(\mathbf{r}) - \mathbf{e}(\mathbf{r}))^{T\cdot}\nabla - \varpi(\nabla^{T\cdot}\mathbf{e}(\mathbf{r})) + (\omega_n^2 - 1)/2\ \right]\ \Psi_n(\mathbf{r}) = 0 \tag{4.2}$$

... where $\mathbf{r} = \{r,\ \phi\}$ and ϕ is colatitude (i.e. latitude measured from polar axis),
$\nabla^2 = \partial_{rr} + (2/r)\partial_r + (1/r^2)(\partial_\phi + \cot(\phi))\partial_\phi - (1/2 + m_{ln}^2)\csc^2(\phi)/r^2\ , \quad \nabla = \{\partial_r,\ (1/r)\partial_\phi\}$

The $\csc^2(\phi)/r^2$ term in equation (4.2) needs to be negative to represent the second derivative from orbital spin in the separated azimuthal factor that has been omitted. Compare

the similar terms in (4.1) where it is positive and is induced by spherical structure and in (5.6) where it is negative and comes from intrinsic spin. The presence of such a negative operator term for either orbital or intrinsic spin always forces positive increase (to reduce spatial oscillatory frequency) via the ∇^2 operator term. Whereas the full equation would need to be solved for these azimuthal factors, here the reduced equation must be used with the respective value of m_{ln} artificially set for any particular modal solution that is sought.

When given in this two dimensional form G_n is a real function describing the nth electron modal form over a half plane $(r, \phi) \in \mathbb{R}^2 \quad r \geqslant 0,\ 0 \leqslant \phi \leqslant \pi$ with its edge on the atomic axis. The full modal form can then be reconstructed up to an arbitrary phase by using the respective magnetic quantum number m_{ln}, spin quantum number m_{sn} and frequency ω_n. Electronic excitations (deviations from the Fermi distribution of electron energy levels without net effect on atomic charge) are then a matter of altered modal amplitudes. Intrinsic spin states arise in a 2-space manifested in relative rotations about the atomic axis of the four component waves constituting the spin factor element (spinoid) and also in orthogonal linear combinations of axially inverted forms of that element (See appendix B).

########

Equation (4.2) can then be split into two equations controlling independent factors corresponding respectively to coordinates of the 2-sphere surface $\mathbf{s}$ in (4.3) and the radius r in (4.4).

$$\left[\begin{array}{r} \partial_{\phi\phi} + \cot(\phi)\partial_\phi - (1/2 + m_{ln}^2)\cot^2(\phi) \\ +\varpi r^2((\jmath\mathbf{v}(\mathbf{s}) - \mathbf{e}(\mathbf{s}))^T \cdot \nabla_\mathbf{s})|_\phi \end{array} \right] G_{sn}(\phi) \ = \ 0 \tag{4.3}$$

... where $\mathbf{s} = \{\phi, \theta\}$, ϕ is colatitude (i.e. latitude measured from polar axis), and $\nabla_\mathbf{s} = \{\partial_\phi, \csc(\phi)\partial_\theta\}$

$$\left[\begin{array}{r} \partial_{rr} + (2/r)\partial_r - (1/2 + m_{ln}^2)/r^2 + (\omega_n^2 - 1)/2 \\ +\varpi(\jmath\mathbf{v}(r) - \mathbf{e}(r))\partial_r - \varpi(\nabla^T \cdot \mathbf{e}(\mathbf{r}))|_r \end{array} \right] G_{rn}(r) \ = \ 0 \tag{4.4}$$

... where $\mathbf{r} = \{r, \phi, \theta\}$, ϕ is colatitude (i.e. latitude measured from polar axis), and $\nabla = \{\partial_r, (1/r)\partial_\phi, (\csc(\phi)/r)\partial_\theta\}$

4.2 Separation of the Ripple Factor

Further separating factor functions on the radius as $G_{rn} = H(r).G_{Hn}(r)$, but omitting magnetic terms for initial simplicity:

$$\left[\ \partial_{rr} + (2/r + \varpi 4\pi Q_A(r)/r^2)\partial_r + |\Gamma(r).H(r)|^2 \ \right] H(r) \ = \ 0 \tag{4.5}$$

$$\left[\ \partial_{rr} + (2H_L + 2/r + \varpi 4\pi Q_A(r)/r^2)\partial_r - (1/2 + m_{ln}^2)/r^2 + (\omega_n^2 - 1)/2 \ \right] G_{Hn}(r) \ = \ 0 \tag{4.6}$$

To maintain steady state the solutions of the radial and latitude factors both require to generate no translative power transfer, so must be in the form of the sum of conjugate

terms with equal magnitudes (producing standing waves with constant complex argument). Thus the paired G and H functions for the radial factor must also have this symmetry. The azimuthal factor for a single mode can (due to its circular continuity allowing phase circulation), and must (to maintain the structure of intrinsic spin) consist of only one of its pair of conjugate complex solutions.

########

We may seek factors $H(\mathbf{x})$ and $G_n(\jmath\omega, \mathbf{x})$ in the wave function $\Psi_n(\jmath\omega, \mathbf{x})$ to allow us to make a separation of factors. This is justified because of the very great difference in frequencies between the factor G_n around the Compton frequency and H having steady real value for the steady state atom. A further separation of a spin factor is also necessary for a complete solution (see appendix B), but for the limited purpose here it can be left implicit via specific terms introduced within the differential operators of equations leading respectively to factors H and G_n. So extending from equation Emode (3.9) to incorporate the factors:

$$\left[\ \nabla^2 + (\jmath\mathbf{v}(\mathbf{x}) - \mathbf{e}(\mathbf{x}))^{T\cdot}\nabla + (\nabla^{T\cdot}\mathbf{e}(\mathbf{x})) + \omega_n^2 - 1 \ \right] \ \{H(\mathbf{x}).G_n(s, \mathbf{x})\} \ = \ 0 \tag{4.7}$$

To show how we identify separable terms in the differential operator the following abbreviated expansion is helpful.

$$HG'' + 2H'G' + H''G + (\jmath\mathbf{v} - \mathbf{e})^{T\cdot}\,(HG' + H'G) + ((\nabla^{T\cdot}\,\mathbf{e}) + \omega_n^2 - 1)HG \ = \ 0 \tag{4.8}$$

We may extract terms in the zero differential order of G_n, and in so doing separate all of the electric divergence, and hence all of the charge, from the remaining wave function factor:

$$H''G + k_h(\jmath\mathbf{v} - \mathbf{e})^{T\cdot}\,H'G + (\nabla^{T\cdot}\mathbf{e})HG \ = \ 0 \tag{4.9}$$

A constant k_h has been introduced to define what fraction of the term in $H'G$ will be incorporated in the Ripple equation, the remainder then being left to appear in the Residual equation here below. (Independent constants of this kind can probably be introduced here for both the electric and magnetic terms respectively.) The choice of this constant affects the resulting form of the ripple function H. But just as we are free to assign this term different values affecting the separation of factors, so we may consider corresponding different families of solutions for the system of equations.

For the study of nascent chemical and photo emission states of electrons we have the opportunity to consider non zero cases of k_h representing "hot" states of the electron field. But these additional terms left behind in the Ripple equation tend through thermal phonic (in molecular and crystalline states) and electromagnetic (in general) emissions to low values for least energy in the thermally relaxed state. So the simpler case of $k_h = 0$ describes the "cold" unexcited steady state (approximating the weakly excited thermal equilibrium state) and so will serve our purpose for the basic analysis here below. Note that this kind of excitation of electrons in the molecular/crystalline state is global to an ensemble of electrons under a common ripple function. It is not the form of excitation that is the basis of Rabi processes and their associated fluorescence.

Writing the equation in full with G divided out gives us a general Ripple Equation common to all electron modes on the given atomic axis. **Eripple**

$$[\nabla^2 + k_h(\jmath\mathbf{v}(\mathbf{x}) - \mathbf{e}(\mathbf{x}))^T.\nabla + (\nabla^T.\mathbf{e}(\mathbf{x}))]\, H(\mathbf{x}) \;=\; 0 \tag{4.10}$$

Some further comment on magnetic effects is given in section 4.4 below. However this also is not needed for the simpler cases of overall magnetically neutral electron fields such as exist in noble gases. They will suit us for an initial study.

4.3 The Residual Modal Equation

We may use the remaining terms after removal of the ripple differential operator terms from equation (4.8) to build a residual equation:

$$HG'' + (2H' + (\jmath\mathbf{v} - \mathbf{e})H)^T G' + (1 - k_h)(\jmath\mathbf{v} - \mathbf{e})^T.(H'G) + (\omega_n^2 - 1)HG \;=\; 0 \tag{4.11}$$

Dividing by H and writing this in full gives a spinless Residual Mode Equation: **Eresid**

$$\left[\begin{array}{l} \nabla^2 + (2\mathbf{H}_L + \jmath\mathbf{v}(\mathbf{x}) - \mathbf{e}(\mathbf{x}))^T.\nabla \\ \qquad +(1 - k_h)(\jmath\mathbf{v}(\mathbf{x}) - \mathbf{e}(\mathbf{x}))^T.\mathbf{H}_L + \omega_n^2 - 1 \end{array}\right] G_n(\mathbf{x}) \;=\; 0 \tag{4.12}$$

... where $\mathbf{H}_L$ is an abbreviation for either of the ripple expressions:

$$\mathbf{H}_L \;=\; \{\nabla H(\mathbf{x})/H(\mathbf{x})\} \;\equiv\; \{(\mathrm{sign}(H(\mathbf{x}))).\nabla \ln |H(\mathbf{x})|\}$$

The outer brackets { } on these expressions are to stress that they must be evaluated before being applied in the equation expression.

4.4 Orbital Magnetics

Although orbital magnetic effects will not be developed in this essay, some comment on the nature of the structure leading to the appearance of quantised magnetic dipoles will help complete the picture here.

The nature of intrinsic spin is different from this orbital kind ... it does not produce simple magnetic dipoles but rather paramagnetic dipoles. The strength of any magnetic dipole produced in that way is proportional to the strength of the applied independent magnetic field which may originate from afar or within the orbital structure (spin orbit coupling) of the same atom. Such intrinsic spin effects will be developed separately herein, and outlined overall in appendix B.

By contrast the principle underlying orbital magnetism is that a given modal bound charge that is not cancelled by oppositely paired magnetic quantum numbers will always give rise to a related fixed magnetic dipole moment depending upon the modal frequency and the value of its magnetic quantum number m_l. This can be understood by recognizing the inherent relationships surrounding the magnetizing mechanism as expressed in EHt equation (2.1) or Echarge equation (2.2).

Considering the circularly symmetrical structure of an electron mode bound to an isolated atom, then any small volume of cisoid matter wave with intensity $|\psi^2|$ gives rise

to charge density $\varpi|\omega\psi^2|$ with an electric field divergence $-4\pi\varpi|\omega\psi^2|$ and, from its atomic azimuthal structure, to a magnetic dipole induction density $2\pi\varpi|m_l\psi^2|$. The ratio of these two densities is always $|m_l/\omega|/2$. Thus any magnetic dipole contributions arising from quantised charges in atomic bound electrons will occur in quantised units of magnetic dipole strength inversely proportional to the respective modal frequencies.

We arrive at a quantal value for that magnetic dipole strength normalised to the Compton frequency, i.e. the Bohr magneton, of $\mu_B = 2\pi q_e/4\pi\omega_C = q_e/2\omega_C$. (Ref A. Mahajan and A. Rangwala. Electricity and Magnetism, p. 419 (1989). Via Google Books) Check the above formulæ.

Generalisation of this relationship to atoms in proximity where the circular symmetry is therefore lost requires proof of its invariance according to principles of conformality of the fields involved. Further algebraic work is needed here.

For cases where electron excitation modifies the modal charges from their individual quantal values (by transferring charge from one electron mode to another) the orbital magnetic dipole contributed will also change slightly. Thus where an atomic species undergoes excitation we may expect small deviations (usually reductions) of the net magnetic dipole from the ground state quantal value. This might be demonstrable as a slight broadening and reduction of splitting of Zeeman spectral lines of say an unpaired isolated outer $p_{\pm1}$ electron under well tuned radiation exciting it to a higher energy level. Does Zeeman splitting depend upon total dipole strength in this way, or is it all proportional? Add a sensitivity calculation here.

5 Charge Ripple Integration

5.1 Separating Structures in the Matter Wave Field

Factors in the Modal Wave Functions

It appears that this subsection might need to be expanded to be more significant overall.

I am trying to subdue the battles between the ideas of harmonics, harmonic functions and weighted orthogonal polynomials in the solutions of the electronic modal wave equation. The structure of the resonances appears to be in amongst that lot. In particular I am wondering whether the azimuthal orbital harmonic has to affect the form of the latitude profile. It appears that it must, but I am not convinced. Hence the need to get a view on the maths notions.

So long as the spatial factors can be orthogonally separated in principle, then the charge can be worked out in one dimension alone. You do not even need to evaluate the separated linear factor G equations to do that. Once they are properly separated you only need the radial equation that contains the cubic nonlinearity, and even that only in a token form involving H within each ripple lobe.

This applies to the evaluation of charges and of topological form only. If you want to compute energies and temporal frequencies, or even the precise modal field shapes, then you need to do full evaluations, at least of the electric field.

The general coupling coefficient that is called the Fine Structure Constant, being involved with charge alone and not energy, can be evaluated without evaluation of the electric field beyond a general simple structure, but does depend upon a stochastic equilibrium only explained by the lower level of the model.

So long as the coordinate dimensionally orthogonal factors separate fully (no remaining cross terms etc.) then it is possible to normalise all of the n dimensional factors except one. If that one carries all the nonlinear structure of the original equation then all the other factors can be replaced by the $(n-1)$-volume of the $(n-1)$-space over which they are integrated. The one dimensional nonlinear integration can be performed in the single remaining dimension alone (the radius for an isolated atom), and solutions to the $(n-1)$ linear factor equations do not even need to be evaluated for this purpose.

So much for the orthonormalisation of the non-radial orthogonal coordinate factors ... we are left to concentrate on the radial dimension. But bear in mind, the main mechanism determining the "meta-thermal" equilibration of states of electronic mode excitations comes from the substrate scattering processes. So we have to introduce a certain amount of further control from the substrate to deal with the charge quantisation in the respective ripple lobes ... thus we still do not have a mechanism based upon Emode alone, in spite of the presence of cubic system structure and ripple form in the electron field.

We can still separate variables for the modes G_n and the ripple function H. We recognise that the modes remain as linear sub-systems requiring only a single coefficient to set the inter-modal relative amplitude of each. So long as the sum of squares of those coefficients

(the one remaining degree of freedom) corresponds to the total charge in the atom then we may study the cubic nature in H separately.

Emergence of Coupling Effects
To quantify the overall electrodynamic coupling strength we need to study the substrate process loop from origination of charge and current through electromagnetic transmission to arrival as coefficients in the matter wave equation. We may recognise that the electron Compton frequency takes its value such as to bring the loop coupling strength to a natural value for that process. It does this by controlling the Compton frequency as a fraction of the Planck frequency in a ratio that, for this model of the underlying dynamics, is controlled by the excess kurtosis of the substrate wave field.

Using this principle by which kurtosis factor establishes the Compton frequency we can infer the inherent relationship of charge origination factor driving the electromagnetic field to the interaction coefficient sensitivity factor. The inverse square law power intensity transmission then completes the model determining the characteristic loop coupling strength. On this basis we may set up the time (frequency) dimension using the Compton (angular) frequency for reference as the unit and thereby accommodate the field coefficients also as having natural units scaled in the simplest possible way. We may then expect the remaining part of the evaluation of coupling strength to consist of an evaluation of the relevant field integrals.

So beginning with the main constants set up in their simplest possible form we may then seek a derivation of the single factor that determines the specific unitless constant relating the universal quantal charge to the corresponding natural unit.

Paradoxical Discrete/Continuous Charge Values
Perhaps the clearest evidence of need for radical review of mechanism within this part of the model (and in the same area of general physics too) lies in the necessity to find a common basis for two independent emergences of the same charge quantum value:

> **1 - A Rigid Discreteness of Atomic Charges**
> Distinct material entities like atoms assume discrete values of charge that are multiples of a unit quantum. We infer here therefore that all microscopic measurements are subject to a quantal nature because the amplification essential to any observation depends upon changes between these discrete states.
>
> **2 - A Thermal Equilibrium in the Fermi Distribution**
> The states of superposed electrons within an atom or molecule are not simply quantised. An evidently thermal process determines Fermi-Dirac statistics in continuum levels of the Fermi distribution of electronic mode states, but with a limit at absolute zero temperature showing the same discrete unit of charge for each of the superposed electronic (Fermionic) modes.

The cubic quantising process proposed above will not, by itself, simultaneously yield these dual emergent forms of the quantal charge unit. It requires another degree of freedom to

be determined within the solution processes of the electronic modal wave equation. This demands that we recognise an additional emergence from the stochastic substrate processes. It is a subtle disjunction of continuity of the wave process arising at the spherical surfaces of zero charge density between the radial ripple lobes.

It appears that at these quadratic zeros of charge density there is a freedom for sufficiently high order derivatives of the matter field intensity in the atomic radial direction to take a step change. (It turns out to be a step discontinuity in the fifth order deviation from the straight line H, i.e. the fifth term in the Taylor series expansion about the zero surface.) This discontinuity imposes extremely little burden of disturbance on the stochastic processes of the substrate ... the zero of the ripple function H makes that possible. But it then also relies upon a process occurring in the substrate system to equilibrate the charge/energy density ratio in the respective ripple lobes to its cosmologically local stochastically determined value of Planck's constant. (Check for precision of the statement in that form.) That appears possible, but details of the mechanism will be developed elsewhere.

Each of the charge quanta then lies within a region separated by quadratic zeros of matter field intensity (and therefore also of charge density) at the enclosing surfaces. Figure 5.1 relates to this, showing how the division of charge by ripple lobes and by electronic eigenmodes are effectively on orthogonal coordinates.

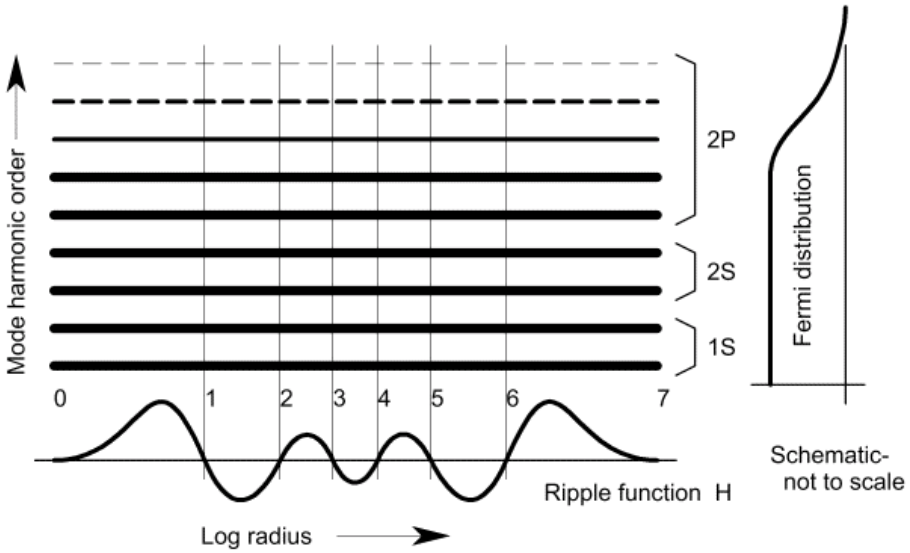

Figure 5.1: **Schematic of Atomic Electron Field (Nitrogen)**

Thus it appears that in completing the electrodynamics model proposed here, effects of the substrate stochastic processes must contribute effects essentially at two points in the overall closed causal loop of the model. The first of these is in the Hilbert transform relationships between matter field intensities and the charges and currents they manifest, and the second is in the appearance of a stochastic charge regulating effect in every solitonic

resonant element of matter. Overall these two effects quantitatively cancel in fundamental terms, leaving the cubic ripple process of our wave equation to determine the way that the zero surfaces in its solutions isolate the quantal charge units.

5.2 Substituting Charge Density

The divergence of the electric field $\nabla\cdot\mathbf{e}(\mathbf{x})$ may be replaced by an expression in the charge density $q(\mathbf{x})$ derived from the individual electron modal variables. Note that negative charge induces positive divergence:

$$\nabla\cdot\mathbf{e}(\mathbf{x}) \;=\; -4\pi q(\mathbf{x}) \;=\; -4\pi\varpi\Gamma_E(\mathbf{x}).\,(H(\mathbf{x}))^2 \tag{5.1}$$

... where $\Gamma_E(\mathbf{x})$ is the the sum of electron residual modal intensities weighted by frequency and ϖ is the original sign of the charge.

From equation Echarge (2.2) we have the charge and current density 4-vector real function $\mathbf{q}(t,\mathbf{x})$. Also we have conditions for unitarity of the oscillatory process as $\mathcal{I}\nabla G_n(\mathbf{x}).\nabla H(\mathbf{x}) = 0$ and $\mathrm{d}H(\mathbf{x})/\mathrm{d}t = 0$. Using these we may express a 4-vector of the residual electron mode first moments (Improve that nomenclature?) as $\mathbf{\Gamma}(t,\mathbf{x}) = (\Gamma_E(t,\mathbf{x}), \mathbf{\Gamma}_M(t,\mathbf{x}))$:

$$\mathbf{\Gamma}(t,\mathbf{x}) \;=\; \jmath\sqrt{\pi/2}\sum_{m,n}(\mathrm{sign}(\omega_m)+\mathrm{sign}(\omega_n)).\mathrm{cis}((\omega_n-\omega_m)t).G^*_m(\mathbf{x}).\Diamond_{\jmath\omega_n}G_n(\mathbf{x}) \tag{5.2}$$

For the simple model we are evaluating here we are interested in only the steady state scalar electric charge density part of $\mathbf{\Gamma}(t,\mathbf{x})$, namely $\Gamma_E(\mathbf{x})$. Because of symmetries between paired modal terms in equation (5.2) the steady state charge density depends upon the modal self products alone ... the cross terms all cancel so long as modal charges are all equal, otherwise there is spontaneous emission ... work this in. So the scalar function $\Gamma_E(\mathbf{x})$ can be extracted and expressed as:

$$\Gamma_E(\mathbf{x}) \;=\; \sqrt{2\pi}\sum_n\left|\omega_n.G_n^2(\mathbf{x})\right| \tag{5.3}$$

From this we obtain the logarithmic gradient of the charge density as:

$$\mathbf{\Gamma}_{LE}(\mathbf{x}) \;=\; \frac{\nabla\Gamma_E(\mathbf{x})}{\Gamma_E(\mathbf{x})} \;=\; 2\frac{\sum_n|\omega_n|\,.G_n(\mathbf{x}).\nabla G_n(\mathbf{x}).\cos(\beta)}{\sum_n|\omega_n.G_n^2(\mathbf{x})|} \tag{5.4}$$

Cubic Nonlinearity

Thus the system is nonlinear. The feedback involved in the modulated Helmholtz form proposed for the ripple equation (4.10) involves a dependently variable Laplacian:

$$[\nabla^2 + k_h(\jmath\mathbf{v}(\mathbf{x})-\mathbf{e}(\mathbf{x}))^T\cdot\nabla + 4\pi\Gamma_E(\mathbf{x}).(H(\mathbf{x}))^2]\;H(\mathbf{x}) \;=\; 0 \tag{5.5}$$

In our physical space-time this admits of two forms of solution. For any $1, N$ dimensional Minkowski space there is a rectilinear/polyhedral scheme, but for the $1, 3$ dimensional space alone there is also a radial/spheroidal scheme. There are interactions with the form of the function $\Gamma_E(\mathbf{x})$, and this couples in the aggregate effects of multiple electronic

modal wave functions. The nonlinear involvement of the ripple via $(H(\mathbf{x}))^3$ introduces a new absolute scale parameter to the system and for radial solutions there arises from it a means for regulation of discrete charge volume boundaries (radial ripple in H), though still not sufficient alone to determine the Fine Structure Constant.

Zero Surfaces of the Ripple Factor $H(\mathbf{x})$
From the electron residual mode in equation Eresid (4.12) we know that $\Gamma_E(\mathbf{x})$ will occur with zero gradient at every zero surface of $H(\mathbf{x})$. We also know that the number of contributory electron mode wave functions is generally greater than the number of ripples in any $H(\mathbf{x})$ factor that we need to consider, whether it be a rectilinear or a radial one. In any case partially excited modes are permitted; it is only the aggregate electron charge that is quantized by the ripple regulating process, not the individual bound modal electrons.

To gain an understanding of how that happens we might try giving $\Gamma_E(\mathbf{x})$ a uniform constant value and look for the form of each of the two kinds of solutions, polyhedral and spheroidal, for that case. We might seek the form of $H(\mathbf{x})$ satisfying $\nabla^2 H(\mathbf{x}) = k.(H(\mathbf{x}))^3$. However it then transpires that solution will only be possible when $\Gamma_E(\mathbf{x})$ takes a form that serves to define and localize the charge distribution. Though we appear at liberty to approximate $\Gamma_E(\mathbf{x})$ rather freely, we cannot afford merely to disregard the result of that topological interaction. In the spheroidal case this then requires $\Gamma_E(\mathbf{x})$ to be sufficiently rapidly convergent at the atomic centre and in the periphery. So long as it meets these requirements we can use equation (5.5) with any of a wide variety of functions in the rôle of Γ_E and get convergent results for H.

It then remains to iterate that kind of evaluation including computation by the spin modified version of equation Eresid that will now be developed at equation (5.6), and with aggregation of electron modal intensities according to equation (5.2) to relax to an overall solution of the radial electron profiles expressed in H and Γ_E as functions of spatial coordinates.

5.3 Spin/Ripple in Spherical Coordinates

Add a note here or somewhere explaining how a spin twist factor necessarily negative in second derivative operator, delivering the $-\cot^2(\phi)/2r^2$ term to allow latitude profile to emerge also contributes a negative constant $-1/2r^2$ from the $\csc^2(\phi)/2r^2$ that acts to increase the temporal frequency.

Add note here explaining that at a given frequency, for a bi-cisoid of unit peak magnitude the second derivative is of peak magnitude equal to that for the second derivative of unit cisoid. Combining bi-cisoid functions as a product, one over time and one over space, would then give us a construct from which to build a picture of the four part compound structure of the spin factor. (Does that really work?)

The following equation is a formulation in atomic centred spherical coordinates of the radial spin/ripple equation as derived from its free electron field form in equation Eripple (4.10) via its charge substituted version in equation (5.5). The form of the Laplacian used is three dimensional. (Note particularly the presence of the $(2/r)\partial_r$ term below where this

coefficient 2 arises from the order 2 spherical surfaces in the 3-space.) But we need only the two coordinates radius r and colatitude ϕ here because we are modeling the isolated atom which always has circular periodic symmetries in azimuth θ.

$$\left[\begin{array}{c} \nabla^2 + k_h(\jmath\mathbf{v}(\mathbf{r}) - \mathbf{e}(\mathbf{r}))^T \cdot \nabla \\ \quad - \csc^2(\phi)/(2r^2) + 4\pi\Gamma_E(\mathbf{r}).(H(\mathbf{r}))^2 \end{array} \right] H(\mathbf{r}) \;=\; 0 \tag{5.6}$$

... where $\mathbf{r} = (r, \phi)$ and ϕ is colatitude (i.e. latitude measured from axial pole),
$\nabla^2 = \partial_{rr} + (2/r)\partial_r + (1/r^2)(\partial_\phi + \cot(\phi))\partial_\phi$, $\quad \nabla = (\partial_r,\ (1/r)\partial_\phi)$

Could we explain that no solutions below Compton frequency could occur at all if the spin were not present? ... well yes ... the original equation has no solutions in simple eigenfunctions below the Compton frequency. It needs the four part compound structure of intrinsic spin for it to be possible.

Why can frequency not continue to drop to absorb charge density effects too? Suppose it is because it would then never converge at the extremes ... remember, frequency has to be common throughout the electron mode.

It might help clarity to expand the expressions as four component vectors (vaguely similar to Dirac) for both G_n and for the intrinsic spin terms in the residual equation differential operator. It appears that H would not need to change, but scalar G_n would then be replaced by a vector denoted $\mathbf{G}_n$.

For reasons explained after equation (4.9) in section 4.2 above, the following will deal only with the simpler but important cases where $k_h = 0$.

Separated Parts of the Intrinsic Spin Factor

The spatial spin factor must be a harmonic function in order not to upset the constraints expressed in the original modal wave equation (3.9) (except at the atomic axis where the wave function goes to zero ... see appendix B). So $\csc^2(\phi)/(2r^2)$ is the always present operator term generating the sum of the four components of azimuthal angular second derivative of the spin factor, and it is introduced via the ripple equation (5.6) only. Using $\csc^2(\phi) = 1 + \cot^2(\phi)$ we may split the $\csc^2(\phi)$ coefficient of spin induced second derivative as $-1/(2r^2)$ in the radial factor differential and the $-\cot^2(\phi)/(2r^2)$ part in the latitude factor. Then the radial equation in re-balancing its solution will induce a radial amplitude factor proportional to $r^{-0.5}$.

These changes of differential due to the presence of intrinsic spin cancel between the spatial dimensions (i.e. constitute a harmonic function spatial factor), and this induces no change to the modal frequency. However the modified radial decay form increases the peripheral rate of decay. This effect is enough to then allow a central attractive nucleus to further increase the peripheral convergence power sufficiently to maintain a stable atomic structure (finite integral out to infinity). Such is impossible without the intrinsic spin present. The resulting atom re-shaped in this way then develops its stable electron modes with their natural frequencies below the Compton frequency. (Indeed, they could not be

stable if their frequencies were not so reduced below the free space limit.)

The Latitude Profile

The combined effects of spherical latitude form and the presence of orbital and intrinsic spin can be treated as a sub-system producing a wave field factor in latitude. It comprises a weighting function that depends upon the azimuthal mode number of the electron mode in question, and the latitude harmonic form consists of any of the orthogonal harmonics (with order index l_c) that comply with that respective weighting. It manifests itself as a fixed real modulation in latitude of all electron amplitudes with a given order of azimuthal orbital spin. We shall handle it here as a function $G_\phi(\phi),\ G_\phi, \phi \in \mathbb{R}$, with ϕ as the colatitude in $0 \leq \phi \leq \pi$.

Extracting the latitude terms from equation (5.6) but leaving behind a constant $-1/(2r^2)$ to be absorbed in the radial form produces the following equation for a latitude factor $G_\phi(\phi)/(2r^2)$. This latitude factor is then used below to multiply the radial ripple functions $G_r(r)$ or $G_\rho(\rho)$ as appear respectively in spherical or log radius coordinates.

$$[\mathrm{d}_{\phi\phi} + \cot(l_c\phi)\,\mathrm{d}_\phi - (1/2 + m_{ln}^2)\cot^2(l_c\phi)]\ G_\phi(\phi) \ = \ 0 \tag{5.7}$$

That removal of the constant $-1/(2r^2)$ from this equation leaves the powers of $\cot(\phi)$ appearing in its coefficients linearly related to the differential orders of the terms. This then supports a simple pair of eigenfunctions. They vary depending upon ϕ (i.e. the equation is non-autonomous), but the equation is still linear in G_ϕ. It is thus possible that G_ϕ can converge to zero at the atomic axial limits of latitude as it must to allow finite mode energy in the presence of the spin factor.

Also having collected all of the terms affecting latitude harmonic factor here we now see that the equation solves for any integer multiplier l_x on the angle ϕ, so we may duly generalise the equation in that respect for higher order harmonic factors in latitude. The characteristic roots of equation (5.7) are then $-(1 \pm \sqrt{3 + 4m_{ln}^2})\cot(l_c\phi)/2$.

Using $\int \cot x\ \mathrm{d}x = \ln|\sin x| + C$ we see that in the presence of spin there exists an eigenfunction solution possessing minimum planar integral of intensity (minimum energy) near the axis as, up to a multiplicative constant:

$$G_\phi(\phi) \ = \ \exp\left(-\frac{1+\sqrt{3+4m_{ln}^2}}{2}\int_{\pi/2}^{l_c\phi}\cot(z)\,\mathrm{d}z\right) \ = \ (\sin(l_c\phi))^{\left(1+\sqrt{3+4m_{ln}^2}\right)/2} \tag{5.8}$$

... where ϕ is colatitude and l_c the latitude mode index
$\phi \in \mathbb{R}, 0 < \phi < \pi; \quad l_c \in \mathbb{Z}, l_c > 0$

The coordinate system required to achieve the above separation of orthogonal factors in this way may yet have to be perturbed from the simple atom centred spherical coordinate system upon which it is based, at least so as to accommodate the latitude profile of the Γ_E functions ... unless electron modal orthogonality is sufficient to avoid such need. The precise form of such transformation will be developed elsewhere. Here we are concerned only that the extraction of such a factor is possible, and thus validates the existence of a

coordinate chart in which the perturbed radial and latitude coordinates remain mutually orthogonal and support equation (5.8).

This transformation would necessitate oblate/prolate form of the spheroidal iso-amplitude contour surfaces of the radial ripple factor function $H_r(r)$. The latitude factor $H_\phi(\phi)$ would then be expressed in the perturbed coordinate system, but alone $H_\phi(\phi)$ is not sufficient basis to define the perturbed chart exactly. Only the existence of $H_\phi(\phi)$ is established, and furthermore subject to having its persistence (as required for a state attractor) conditional upon other factors that permit the stability of the system.

Autonomy and Symmetry in the Ripple

If the differential equation is to yield behaviour dependent upon integrated charge without regard to radial scale it would need a form of autonomy (non-dependence upon the independent variable within the form of the differential). From the above ripple differential equation (5.6) we can get a (differentially) autonomous version, and up to a scaling constant there is only one such possibility. This is by mapping it onto the log of the radius as the new independent variable.

Logarithmic Radial Coordinates

To bring out some symmetries we transform to log radius $\rho = \ln(r)$ so that:

$$\mathrm{d}\rho/\mathrm{d}r = 1/r\ , \qquad (\mathrm{d}\rho/\mathrm{d}r)^2 = 1/r^2\ , \qquad \mathrm{d}^2\rho/\mathrm{d}r^2 = -1/r^2\ , \qquad r^\zeta = \exp(\zeta\rho)$$

$$\mathrm{d}_r H(\rho) = (\mathrm{d}_r\rho).\mathrm{d}_\rho H(\rho) = \mathrm{d}_\rho H(\rho)/r$$

$$\mathrm{d}_{rr} H(\rho) = \mathrm{d}_r\left((\mathrm{d}_r\rho).\mathrm{d}_\rho H(\rho)\right) = (\mathrm{d}_r\rho)^2(\mathrm{d}_{\rho\rho}H(\rho)) + (\mathrm{d}_{rr}\rho)\mathrm{d}_\rho H(\rho) = (\mathrm{d}_{\rho\rho}H(\rho) - \mathrm{d}_\rho H(\rho))/r^2$$

... and substitute these in the original equation (5.6):

$$\left[\begin{array}{c} \partial_{\rho\rho} + \partial_\rho - 1/2 + \partial_{\phi\phi} + \cot(\phi)\partial_\phi - \cot^2(\phi)/2 \\ +4\pi\exp(2\rho).\Gamma_E(\rho,\phi).(H(\rho,\phi))^2 \end{array}\right] H(\rho,\phi) \ = \ 0 \tag{5.9}$$

To improve graphical depiction of the ripple function an exponential factor can be combined with the variable as $H = \exp(-\rho)\Lambda$, and using the differentiations ...

$$\begin{aligned} \partial_\rho(\exp(-\rho).f(\rho,\phi)) \ &= \ \exp(-\rho).[\partial_\rho - 1]\ f(\rho,\phi) \\ \partial_{\rho\rho}(\exp(-\rho).f(\rho,\phi)) \ &= \ \exp(-\rho).[\partial_{\rho\rho} - \partial_\rho + 1]\ f(\rho,\phi) \end{aligned}$$

... (Something wrong here?) we then have a Symmetrical Log Ripple Equation:

$$\left[\begin{array}{c} \partial_{\rho\rho} - 1/4 + (\partial_\phi + \cot(\phi))\partial_\phi - 1 - \cot^2(\phi)/2 \\ +4\pi\exp(\rho).\Gamma_E(\rho,\phi).(\Lambda(\rho,\phi))^2 \end{array}\right] \Lambda(\rho,\phi) \ = \ 0 \tag{5.10}$$

The only non-autonomous part in this ripple equation operating alone is introduced by the independent choice of positioning of the Γ_E function along the ρ axis. An absolute solution for the electron field result cannot arise until the residual equation is also brought in to complete the loop causal structure of the quantizing mechanism and a stochastic process is introduced from the lower model to drive charge quantum equilibria.

5.4 Convergence of Integrals

In regions both close to the origin and in the far field the value of H becomes vanishingly small so that the cubic term becomes negligible with respect to the others. Remembering we are dealing here only with cases where $k_h = 0$, this leaves equation (5.6) in a constant form of the Helmholtz equation and thereby determines the necessary form of the field in both of those regions, and indeed also in the region of approach to zero intensity that stretches all along the zero axis of the atomic system. (Here we shall work in spherical coordinates of (5.6) but could do an equivalent thing in the log coordinates of (5.9).) Recognizing the spherical field geometry in these extreme regions of the equatorial mid-plane, the asymptote is $k_R r^\zeta$ where ζ satisfies $\zeta(\zeta-1)+2\zeta-1/2=0$ so that $\zeta=(-1\pm\sqrt{3})/2 \approx 0.366,\ -1.366$.

The resulting amplitude factor H, which includes the real radial part of the spin factor, must then be asymptotic to the 0.366 power law in the region of the atomic centre and to the -1.366 power law at the periphery. These asymptotes are consistent with convergence of the wave function in these regions to zero and of the volume integral of intensity to a finite value.

The Form of the Ripple

Considering the single ripple (half cycle) solution, it would appear that there is only one point in the $\ln(r) : \ln(\sqrt{rH})$ plane at which intersection of these asymptotes corresponds to convergence of the function onto these two asymptotes simultaneously at origin and far field regions respectively. We then need to establish where that point is.

With the synthetic electron charge distribution used in the initial exercise here we shall force the symmetry to the point corresponding to unit radius in the natural units (zero in the log plot). When the nuclear charge is introduced the resulting Coulomb field will then provide the basis for a practical solution of the residual equation taking the charge cloud nearer to the atomic centre. The nuclear Coulomb field (the peripheral harmonic and therefore non-divergent part of the nuclear electric field) acts to reduce the electron modal energies. This allows finite length of the matter waves for the electronic frequencies below the Compton frequency that are observed in ordinary atoms.

There are alternative solutions of the same equations in absence of a nucleus. These are without intrinsic spin for frequencies just above the Compton frequency as the free electron field. Also we must presume there to be solutions around much higher Compton frequencies than the electron Compton frequency, these appearing to be the structural elements of the heavier leptons with short lifetime called the “muon” and the “tauon”. Try to give ripple structure description here. But throughout all such different forms of wave solutions quantum unit charge definition remains unchanged because it depends ultimately upon the same underlying stochastic equilibration process.

The solutions of these non-autonomous and nonlinear differential equations can be viewed as a concatenation of many small sections each of which approximates to the solution of a linear differential equation. This is achieved by changing the coefficients of the equation as a function of the independent variable (the radius). Thus we may think of the solution

as built from segments or splines, each associated with a short segment of the radius.

The solutions are anyway asymptotic at the extremities to simple power law form (to exponentials in the log radius transformation) because there, being so small, the ripple function loses its dependence upon the cubic term. Where the cubic term is significant it is then still possible to recognize the symmetry of these effects about a centre point so long as the function can be expressed through transformation as the solution of an equation that is autonomous in everything but the cubic term.

Sufficiently Convergent Modal Form

It is true that the gradient of the function $\Gamma_E(r)$ is constrained by the electron residual mode equation to attain a constant value (zero gradient) at all points where $H(r)$ is zero, and that clearly includes the origin and far periphery. However, so long as the function $\Gamma_E(r)$ converges with a sufficiently high order then it may also take zero value at these limits. When that is so the whole issue of convergence at the limits becomes dependent upon the product $\Gamma_E(r).(H(r))^2$ rather than either of these functions alone.

It is easier to see these relationships in the equations as transformed into logarithm ρ of radius r, and also when the radial intensity gradient that is natural to the spherical solution is divided out to produce clarity of ripple symmetries that are present for discrete charge cases.

Thus we may see that so long as the log transformed version of $\Gamma_E(r)$ has sufficient strength and exponential order of convergence at the extremities then it may assume zero value at the limits. Ultimately evaluation of the residual equation will prove the point, but as a first view of the situation we may use a synthetic function with these properties to see how the ripple function then behaves. The following check shows that the ripple process produces the expected independence in its control of discrete and quantal value. We may expect that to be apparent even in this synthetic situation. Being synthetic in this way it also serves usefully to stress how independently effective is the quantizing effect.

6 Quantization

6.1 Ripple Charge Invariance

A Working Conjecture
As a working conjecture (that is in urgent need of checking and proof) it is posited here that for the isolated atom we need only study those solutions of the modal equation (3.9) for electronic modes that can be expressed as products of orthogonal factors separable by the spherical coordinates, namely radius, latitude and azimuth, about a common centre and axis of the atom.

> Note in passing: It is also expected that extension to deformed atomic structures such as in molecules and crystals will be possible using conformal deformation of the spherical surfaces under three dimensional transformations of the respective atomic fields, but such extension will not be dealt with in this essay.

A Radial Process for Quantisation
Under the above conjecture we expect the cubic process of charge quantum subdivision to involve only the radial coordinate. Thus the separated nonlinear part of the process represented in the Ripple equation (5.6) can be studied in the single radial dimension, leaving all other structure as a linear part in the residual equation, though still amenable to further separation of factors and terms therein as may be needed.

Noting that we have derived two separable factor equations as a basis for solution of our original matter wave equation, it is intuitive to expect there may be need for additional constraint on the possible solutions for the conjoined equations. The residual equation as expressed in (4.12) or (5.6) is linear and so provides no definition of absolute amplitude for its variable G_n. It may be necessary to normalise the overall result to match the respective Fermi levels for the G_n amplitudes, or perhaps to other levels to deal with excited atomic states. However, when these results are then applied to evaluation of charge or current densities it will be necessary to use a corresponding value of H in order to give absolute value to the product $G_n.H$ as it contributes to the aggregate electron intensity expressed as Γ_E in equation (5.3).

6.2 A One Dimensional Test of Topology

In the circularly symmetrical atomic wave equations there are two coordinate variables r and ϕ. Therefore for initial simplified reasoning we would like a one dimensional approximation that will retain the radial topological features we wish to explore. This can be achieved by setting $\phi = \pi/2$, i.e. the $(\partial_\phi + \cot(\phi))\partial_\phi$ term can be ignored.

Evaluation of the One Dimensional Model
An Electron Radial Ripple Amplitude graph can be obtained by integration of the Radial Spin/Ripple Equation (5.6) along the radius from a point near the axis to a peripheral

point. Inclusion of the $\Gamma_E(r)$ factor as a function of radius is straightforward and yields the following form in Figure 6.1 for the ripple function $H(r)$ as transformed into a function of logarithm of radius. This example is for ten electrons (Neon).

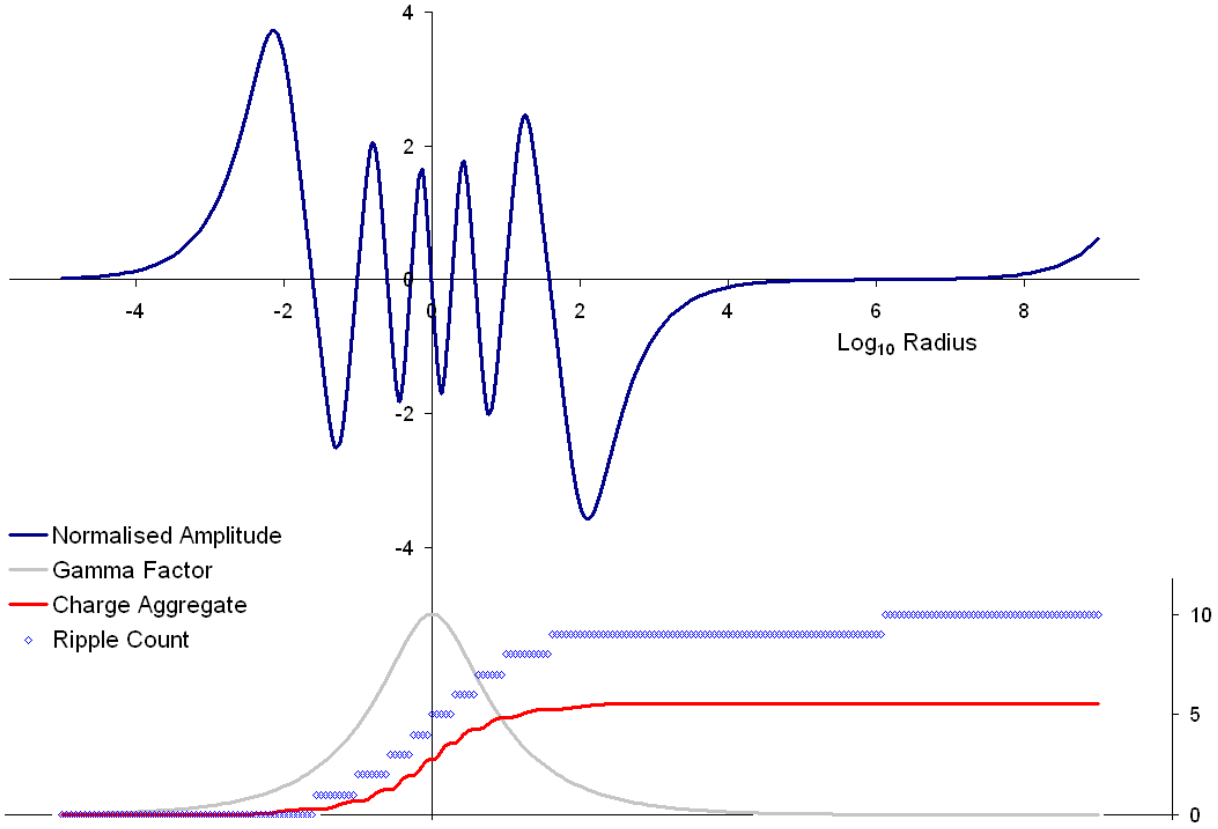

Figure 6.1: **Electron Radial Ripple Amplitude**

The unit of length for the radius in this diagram is the natural unit for electrodynamics, namely the ratio of the speed of light to the angular Compton frequency $c/\omega_C = 3.861590654 \times 10^{-13}$metre. The axis is transformed to log of radius and the amplitude is normalized by multiplication by square root of radius. This produces the above evident symmetry of the ripple so long as the left hand boundary condition is set to one of the feasible discrete values (a slight offset from the discrete result is left present here ... see below). In spite of that apparent symmetry the overall form of the electron field intensity tapers toward the periphery by a further factor inversely proportional to the radius, and that has been divided out from this result. We may assign the influence bringing about that effect as part of the electron intensity profile function $\Gamma_E(r)$.

The above result was integrated in constant proportional steps of radius, relying upon double precision floating point to deal with the scaling of computation. It was then subsequently transformed to the above graph in the process of data output. A similar result might more naturally be considered in the context of the Symmetrical Log Ripple Equation (5.10) that is given above. There this form of function is more evidently the direct outcome

of its integration so long as the reciprocal of radius factor (explicit in that equation as the factor $\exp(\rho)$) is included in the transformed version of the aggregate electron intensity profile function $\Gamma_E \exp(2\rho)$.

Ultimate Peripheral Divergence

The ripple function in the diagram was evaluated with a deliberate very small excess (0.14 p.p.m.) of charge over the discrete value required for exact balance in the periphery. This was achieved by adjustment of the initial condition determining the divergent form at the left end of the log radius axis. It demonstrates the divergent form at the right. This provides the basis for a mechanism of self adjustment to the discrete quantal level of soliton charge. No continuous computational or physical process could ever provide exact balance, so this peripheral divergent structure is a necessary vehicle for sensitive detection and feedback of inexact quantal state. Replacing the excess of charge by a small deficit the amplitude divergence on the right of the graph would occur with the opposite sign.

The regulatory process remains yet to be studied, but given the convergence at both atomic centre and periphery then the ripple function can be treated as a kind of nonlinear eigen function. The ripple function has power of divergence fixed at $\pm\sqrt{3}/2$ at its extremities (in the log plot). So when it is multiplied by an exponentially convergent factor with sufficient convergent power of exponent the product will in any case converge at both extremities. Still the system promises a strong sensitivity to sign of the offset from discrete charge as the sign changes.

This ripple amplitude was evaluated with a synthetic residual intensity profile for the function $\Gamma_E \exp(2\rho)$ rather than that corresponding to an actual electron field for neon (we do not yet have one). The radial function used was a hyperbolic secant in the log radius centred on zero (natural value for unit radius). That gives it an exponential convergence at the extremities. The actual form of a real version of this function would have steps in its flanks with flats corresponding to the charge density zero surfaces.

Independence of Ripple Topology

This evaluation omits the effect of the electric field from the nucleus. (In this respect there is a strong contrast between this model and the Schrödinger equation where no such solution would be possible.) When added it will re-position the electron charge about the nucleus, and the proper inclusion of that effect will require a good appreciation of, and therefore an evaluation of, the individual electron mode wave functions. Nevertheless the quantizing effect of the ripple depends, via the divergence of the electric field, only upon charge density. Such rearrangement of the electron wave function may affect the stability of a given modal form and even destroy its viability, but that will not alter the quantal value to which the charge is regulated in its equilibrium state, be it stable or not.

It is an ongoing subject of the research to check the validity of the latter assertion. Once proven it may be used to help speed convergence of wave function integrations.

6.3 Pauli Exclusion Principle

Issues surrounding the Fermi distribution and origins of the Pauli exclusion principle need to be dealt with. A full account is best framed in terms of the stochastic substrate model and that will not be included here. Some notes about this are given in appendix G.4. For purposes of evaluating the wave model expounded here the conventional form of the Fermi distribution may be invoked as means for determining the charge in the respective electronic modes.

7 Phenomena

7.1 The Critical Evaluation of this Wave Model

The explanation of spin filtering phenomena that are reported from the more elaborate forms of the Stern Gerlach experiments currently forms the crux of the evaluation of this model (See section 7.7 below). If the spatial resolution into evidently quantal results is treated according to the rationale of this model then there has to be what is effectively an amplifier manifested in the underlying wave process that is capable of doing that. For the equivalent sorts of effects obtained with electromagnetic radiation (with photons, as they say) that idea is not difficult to support and explain using the ideas of reciprocal couplings, but for the Stern-Gerlach filtering experiments it drives the requirements placed upon the matter wave model to its full stretch. The outline of explanation exists but further verification will be needed.

Apparently Bohr is quoted as referring to what I would call quantal resolution as involving "irreversible classical amplification". That in spirit is sympathetic to the ideas proposed here, but the processes that form the basis of amplification are themselves subquantal, and therefore not in every sense what we might choose to call "classical". Perhaps in line with the idea of an underlying deterministic process model with emergent quantal observation the term "irreversible amplification" would be better, though elsewhere [AB2004] I refer to it as "unilateral amplification" in line with the engineering usage.

7.2 Lorentz Accelerative Forces

Amongst the many kinds of phenomena that must be properly accommodated in our model the Lorentz forces of the electromagnetic field acting upon charged matter are perhaps the first to be considered. These forces are well known macroscopic effects, yet in conventional physics they are characterised as fundamental relationships involving coupling coefficients with no obvious origins and invoking the idea of mass (as complementary to force in achieving kinetic energy) also without a clear basis in fundamentals. Here we take care to demonstrate that the accelerations that occur in our wave process match those observable in physical systems. Then the extension to ideas of mass and force are introduced in a secondary way subject to additional concepts of how we might spatially partition the overall wave solution.

Electric Field

We may begin with the accelerative tendency of an electron matter field in the presence of a static electric field. This is dealt with conventionally in the Schrödinger and Dirac equations using the potential energy notions of a Lagrangian model based upon potential energy of an electron particle in an electric field. To do that requires that the coupling from field to particle be pre-defined in terms of charge and field strength (Lorentz forces). Here we instead study how the perturbation of the electron modal wave equation by an externally applied electric field acts to alter the wave solution and induce in it an accelerative motion if

unconstrained, or a constantly twisted phase (implying flow of momentum in conventional terms and thereby force) if it is constrained.

Recalling equation (3.9) Emode:

$$\left[\nabla^2 - \mathbf{e}(\mathbf{x})^T.\nabla + \jmath\mathbf{v}(\mathbf{x})^T.\nabla + (\nabla^T.\mathbf{e}(\mathbf{x})) + \omega_n^2 - 1 \right] \Psi_n(s,\mathbf{x}) \;=\; 0$$

If a term in uniform electric field $\mathbf{e}_\Delta$ is introduced to the mode equation then the electron wave function Ψ_n suffers a multiplicative term Ψ_Δ developing according to:

$\Psi_\Delta(t,\mathbf{x}) = \mathrm{cis}(\pm\omega_\Delta(t,\mathbf{x}).t)$

$\omega_\Delta(t,\mathbf{x}) = \mathbf{e}_\Delta(t,\mathbf{x}).\nabla \ln(\Psi(t,\mathbf{x}))/2$

Relate this to acceleration here.

Magnetic Field

More text.

Inherent Electric Gauge Freedom

In defense of our radical modeling we may heed a certain anomaly that arises in the conventional models, namely that the energies involved in high degrees of electric potential arising in nature (e.g. thunder clouds) are inconsistent with the invariance of the chemistry over such differences of potential. Conventionally this is overcome by prohibiting the modeling of such matter fields on a common basis of coordinates. Each electron must be analysed under potentials effectively related to that at its centroid. Thus the conventional models cannot represent a unified causality in this respect (as also in several other respects).

Instead of electric potential we here use only electric field (that is potential gradient in the static case) as the basis of coupling in the equation. This will deny the existence of any absolute effect of potential, and in particular will challenge any literal existence of the electric Aharonov-Bohm effect (see below section 7.11) [AB1959], [AB1961].

7.3 Orbital Magnetic Moments

For the circularly uniform orbitals with $m_l = 0$ (these are S-modes, where there is no net circulatory imaginary derivative of the complex variable) there is no circulating current and hence no magnetic dipole, notwithstanding the presence of spin. Spin produces effects similar to a para/diamagnetic dipole (responding to an applied magnetic field) and magnetic coupling interactions too, but, contrary to much belief resulting from orbital particle electron models, there is no constant magnetic dipole. Refer to appendix B for a wave model explanation of why it would work that way.

In contrast for any $m_l \neq 0$ the spatial derivative has a circumferentially pointing term about the atomic axis, and the product of the field terms according to EHt equation (2.1) and Emode equation (3.9) is then real, hence producing circulating current. Thus for these $m_l \neq 0$ cases a magnetic dipole that points axially is always produced, though it will be cancelled when the modes are oppositely paired.

7.4 Normal Zeeman Effect

Here is an outline description. It is rather complicated, so for further clarification please refer to Wave Topology of a Spin Mode [ABoWT]. Much work remains to be done in vindicating and verifying quantitatively these spectral effects.

In so called "normal" Zeeman interactions a plane parallel static magnetic field acts to modify the resonant frequencies of the electron modes, and these shifts become apparent through the electromagnetic absorption and/or emission effects accompanying the transfer of energy between one of these composite modes (in isolation if the so called "anomalous" spin effects are to be avoided) and another with a ± 1 different principal quantum number and opposite spin within the same atom. The coupling relies upon the product of the two wave functions having a dipole form so that they can interact with the electromagnetic field, typically at optical frequency corresponding to the modal absolute frequency difference. That dipole coupling necessity gives rise to the existence of the transition selection rules.

When an atom is immersed in a uniform magnetic field then for every electron mode the operator term $\jmath\mathbf{v}(\mathbf{x})\nabla\Psi_{\mathbf{n}}$ when integrated in its effect over the volume of the atom produces a complex result from this magnetic interaction divergence term. The imaginary part of this term is (at least approximately) independent of the direction of the magnetic field relative to the atom. When added into the differential operator the imaginary part of this term produces a fixed sign of frequency shift, and in terms of absolute frequencies that has opposite shift effect upon wave function terms with opposite signs of complex frequency. Since each composite spin factor contains two wave elements with opposite signs of frequency the system of each electron mode can only accommodate the magnetic field whilst still satisfying the modal wave equation by rotating about the atomic axis at a rate proportional to the strength of the magnetic field.

The effect of this upon the strength of the dipole coupling product between two electron modes is to modulate it at a frequency that is the product of the rotation rate and the order m_l of the azimuthal harmonic. Since these couplings only occur between electron modes with opposite spins the rotations of a pair of coupling modes are oppositely directed. Thus the beat frequency this produces in the coupling dipole strength and therefore also the shift in electromagnetic interaction frequency corresponds to the sum of the azimuthal modal orders involved.

The presence of any azimuthal harmonic order $m_l \neq 0$ also produces local interaction due to its imaginary circumferential relative gradient. This suggests a real overall contribution to the differential operator. It has a coupling that varies with the latitudinal orientation of the static field to the atomic axis. This has the effect of shifting multiplicatively all of the resonant frequencies within each atom and in proportion to their respective values of m_l, but by differing amounts in the differently oriented atoms. Since it has proportionate effects on the oppositely signed frequencies within any given mode it alters the rotation rate only in proportion to the shift (say an optical frequency) as a fraction of the absolute electronic frequency (near the Compton frequency). But that is a very small fraction $\approx 10^{-5}$ of the dominant rotation rate caused by differential effect from the imaginary term so we would expect no more than a slight broadening of the spectral

lines due to the distribution of atomic orientations.

7.5 Anomalous Zeeman Effect

Each individual excited electronic mode has a spin factor structure with a 2×2 division into four interlocked non-orthogonal parts with precisely balanced intensities. (See appendix B.) Only when all four parts are taken together does the mode satisfy the modal wave equation. These parts occupy respective combinations of positive, negative or zero frequency, and opposite twists or none in the azimuthal plane. The frequency/twist pairings in the four parts within such a composite mode are $++, 0-, 00, -0$, or any of the three other frequency/twist sign reversed versions of this. Thus within any one composite mode the two frequencies are of opposite sign and the two twists are of opposite chiral type.

In spite of the absence of a spontaneous magnetic dipole for the $m_l = 0$ cases, still when immersed in a magnetic field then according to equation Emode (3.9) the magnetic coupling term $\jmath\mathbf{v}(\mathbf{x})\nabla\Psi_\mathbf{n}$ produces an integrated imaginary bias of the differential operator that results in an opposed marginal change in absolute values of the two oppositely signed frequencies involved as parts of the spin factor. Similarly to the case for the normal Zeeman effect, this forces the mode concerned to rotate about the atomic axis, and that rotation takes the opposite directions for modes with opposite spin senses.

Does this process lock to a minimum energy alignment of an anti-spin pair below a critical temperature?

7.6 Stark Effect

Effects similar to those in the Zeeman effect are introduced by the electric field, but in rather complicated ways. Unlike the cross product interaction involved in the Zeeman effect above, they enter via a perturbation of the form $\mathbf{u}(\mathbf{x})\nabla\Psi_\mathbf{n}$ and because of diametric symmetry there is no first order effect in an isolated atom. But the asymmetrical product tends to distort the modal forms through induced finite phase shift (the origin of Coulomb force) whilst not moving the neutral atom.

It is only with stronger fields that a difference comes about, presumably via induced asymmetry of the modal structures producing differing effects of net frequency change respectively in the different electron modes. As would be expected, the effects then appear as having a quadratic dependence upon the electric field strength, but there are exceptions to that quadratic relationship, notably for hydrogen. Work is needed to evaluate these effects in terms of wave modes.

7.7 Stern-Gerlach Effect

This is an effect observed in a beam of electrically neutral atoms having an unpaired S-mode electron in each atom when the beam is passed through a tapered magnetic field.

There is no magnetic moment produced by an S-mode electron regardless of whether it is isolated (i.e. not member of a spin pair), nor even under an applied magnetic field.

There is just no net circulation of current. However, an effect similar to that of magnetic moment can arise for an electron bound in an atomic unpaired spin mode if the magnetic field is tapered. Then such electron modes generate effects as of either para or dia magnetic dipole moment depending on spin sign (the Stern-Gerlach effect). The effect depends upon chirally handed phase gradient induced in the wave function along the axis of the tapered field. This phase twist is induced by the imaginary gradient in the differential operator resulting from the taper in the magnetic field strength. Unlike the case for a magnetic dipole, this effect is independent of the orientation of the magnetic field in relation to the atomic axis.

When a phase twist exists diametrically across a mode then there is a finite phase velocity across that diameter, and that leads directly to a group velocity, and hence bodily motion of the modal field as a whole in order to satisfy the modal wave equation. The equilibrium can then only be found by the electron mode in question biasing its position away from the centre of the atom so as to produce a converse phase gradient to maintain the atom as a coherent whole. Such a distortion of the modes occurs for a uniform magnetic field too, but since it cancels the phase twist there is no net accelerative effect. With a tapered magnetic field that compensation cannot become complete and an accelerative effect remains.

Any asymmetry of overall phase twist in the atom produces a net acceleration. By treating the electron as a separate entity this may alternatively be evaluated in terms of the force needed to hold the offset electron in the atom, but care is then needed in evaluating the development of the joint charge distribution. The electron is not, in the simple sense, distinct from the atom.

This mechanism produces a para or diamagnetic effect depending on spin sense as though there were a magnetic dipole present, but the direction of the dipole always appears as collinear with the magnetic field and oriented in the same sense depending only on sign of spin. That phenomenon cannot be properly modelled by any kind of magnetic dipole.

The para/diamagnetic effects of spin arise as an additional effect regardless of the azimuthal order m_l and consequent constant magnetic dipole of the mode concerned.

Regarding the 2×2 state space needed to represent the behaviour of a spin bearing particle, as explained above, the state of an atomic bound electron wave mode involves a spin factor with components taken from a set of four possible orthogonal element wave functions, each of these also being a composite of four equally intense parts. The forms of equilibrium between these former four, in a pattern of 2×2 subdivision by sign of frequency and chiral sense of azimuthal twist, have two degrees of freedom for fluctuations about equilibrium and this provides the basis for the mechanisms originally expressed formally by Dirac, and normally represented by the Pauli 2×2 matrices.

The presence of a magnetic field induces a rapid precession in the abstract state space of these 2×2 pairs. When a tapered magnetic field is applied then the abstract state space axes of these paired modes rotate oppositely at a much lower and variable frequency depending on the taper, but high enough to produce several cycles of this second gyration in the time of flight through a single Stern-Gerlach magnet. This brings about destruction of spin correlation between orthogonal successive Stern-Gerlach spin filters.

For further details refer to "Wave Topology of a Spin Mode" [ABoWT].

7.8 Wherefore Magnetic Monopoles?

In this model the electron is undeserving of its identity as simply an electric charge particle; it is equally involved in both electric and magnetic fields. The possibility of a magnetic monopole density does not arise because the nature of the magnetic effect arises originally as a vector, whereas electric charge appears as monopole because it is a scalar product of scalars. The symmetry between electric and magnetic effects that is normally stressed in the Maxwell equations is in this sense redundant and even misleading.

7.9 Electromagnetic Emission and Absorption

The basic case of transfer of energy between an electromagnetic radiation field and an electronic modal system involves the de Broglie wave structures of two spatially overlapping electronic modes (involving a common spin axis) and the electric and magnetic fields of the radiation. The rôle of the radiation involves its wave gradient, and because of that the simple case is where the effective modal diameter of the roughly spherical electron is less than half a wavelength of the radiation field, so that covers the cases up to X-rays around 10^{18}Hz. Lower frequencies such as those of light (around 0.7×10^{15}Hz) are capable of interactions with the thousands of times higher frequencies of the electron modes near to the Compton frequency (1.2356×10^{20}Hz). Through their wave function products they are only involved with respect to the differences (produced from products between modes with like spins) of these high frequencies.

The strength of this basic form of coupling depends upon the approach to a triple frequency coherence between the two electron modes and the radiation. The energy fluctuation is called a Rabi process, and it can be characterised in a three dimensional state space of energies and phases called a Bloch space. The magnitude of the peak energy fluctuations involved varies approximately in inverse proportion to the difference from the ideal frequency coherence condition. Over one half of the phase relationship cycle of the waves at the triplet of frequencies there is a charge conserving energy transfer from the electron mode with higher absolute frequency to the that with the lower with a net increase of energy in the radiation (stimulated emission), and for the other half of the phase relationship the energy flows are the other way (absorption).

There is also one more aspect of phase relationship controlling the reactive transfer of energy between the modes (treating the radiation as a mode too), and hence pulling of the electron frequencies and phase shifting the radiation occurs. In all cases the instantaneous rates of transfer into each of the three wave modes involved is proportional to its signed frequency, and there is no net gain or loss of energy from this triple system ... nothing else is involved. There is no such intuitive quantitative model of interaction based upon the idea of the particulate electron in an atomic orbit.

7.10 State Transition Selection Rules

For coupling like this to be possible at all the two electron mode fields must have a dipole distribution of the amplitude of their field product and the axis of that dipole must lie, at least partly, in the transverse field direction of the radiation. The possible alignments of electric and magnetic fields with this product can be evaluated by starting from equation Emode (3.9). The coupling can only happen for certain pairs of modes that have a difference of one order in certain groupings of their spherical harmonic modes.

Think of the case where they have a difference of one cycle azimuthally and are the same order axially, that corresponds to a difference of ± 1 in the azimuthal mode order, none in the axial order and therefore demands that for matching radial orders there shall also be a difference of one in their principal quantum numbers. We might also consider the alternative in which they have identical azimuthal orders and differ by one axially, and again a difference of one would be needed in the principal quantum numbers to accommodate this. The former of these is a valid coupling but the latter may not be. (I need to do further checking of that specific point, but the basis is pretty clear.)

Thus with this model the possibility or not of electromagnetically induced excitation and coupling between the various mode pairs is made clear in a rather simple way, and through the spatial moment of intensity distribution of a mode pair product the strength of its coupling can be calculated. It is not likely that the nature of these processes could be either grasped mentally or computed with such alacrity using a particulate notion of the electron. What is more we can take such a model farther, such as by suggesting that for higher frequency X-rays it is then presumably possible to reckon a different set of selection rules for couplings via higher order spatial moments of mode pair product cases.

This model for establishing possible state transitions and their coupling strengths can be extended to the substantially different geometries of the electron fields of molecules and crystals.

7.11 Aharonov-Bohm Effects

The magnetic Aharonov-Bohm effect is commonly explained in terms of the existence of the magnetic vector potential field underlying the magnetic field. Here we use the notion of the vector potential as a field, but do not need to assign it the status of having a locally detectable existence beyond the value of its curl.

Whereas the conventional argument uses the change in phase along two independent paths as evidence of their respective differing strengths of the vector potential, the effect is here explained as a field integration of differential phase over the entire space that includes and intervenes between the two paths under comparison. After all, the Maxwell description of these fields maintains that the magnetic field is exactly representable as having no divergence, so must be precisely equivalent to the curl of another field. So it matters not whether we bother to claim the existence of this other field so long as we maintain precision of the field integrations involved so as to dispose of the arbitrary possible phases. Even though the electron beams pass through regions of indefinitely weak

magnetic field, still the associated spatial integrals of the electron field that lead to the respective phases must be taken into account.

Indeed, the means for sensing phase along the respective paths is always a pair of coherent free electron beams, and it is the notion of this coherence as being continuous across the whole of the intervening space that allows us this account.

Rather than seeking further enlightenment about gauge invariance of these fields so that particles can meaningfully exist in them, the upshot of this way of understanding the nature of the electron beams is that we need to grasp better the extent to which electrons are to be seen as comprising distributed wave processes.

It remains to be seen whether there is or needs to be a similar basis in distributed electron fields that can support the notion of the (as yet un-demonstrated) electric Aharonov-Bohm effect for free electron beams. It seems unlikely.

8 Prospects and Further Work

8.1 Feasibility

An intuitively attractive electrodynamics model based upon waves in a continuum does indeed appear to be possible. Starting only from notions of the nature of eigenfunctions, there are phenomena illuminated by this model that seem impossible to depict properly with particle based models. The non-radiating bound electron and the absence of strong up conversion through mixing of fields in the oscillatory steady states (UV catastrophe) are particularly powerful examples of this.

The introduction of nonlinearity through the couplings between otherwise linear equations produces a model that has attractive options for approximation because of the tendency for changes of state ... the Rabi processes ... to occur at frequencies much lower than those of the electronic modal eigenvalues ... near the Compton frequency. Thus a weakly or slowly perturbed linear system of high frequency electrons is a simple and valuable approximation. Its applications include the very interesting cases of Rabi dynamics using the Bloch sphere state space, both on the sphere (e.g. quantum computing qubits) and within it (e.g. laser dynamics).

The ability to account for intrinsic spin in terms of factor functions in wave topologies is probably unique to this sort of model. It offers a promising path to better comprehension of the puzzling nature of this salient feature of quantum mechanics.

The spontaneous quantizing ability of this model is elegantly demonstrated by the existence of a ripple solution factor synthesized from a symmetrical pair of modes with real characteristics diverging in opposite directions along the atomic radial line. It is only certain discrete values of the charge that allow such a solution to converge simultaneously at the axis and the periphery of the atom. Stability and actual charge values of these quantal states remain to be studied.

The model offers an additive superpositional continuum structure in place of the usual multiple disjoint Hilbert configuration spaces for multiple electrons. It is therefore capable of improved computational efficiency because it can replace the chaotic nature of the many body collisional problem with a multidimensional relaxation problem in smooth functions. Though Density Functional Theory is set up to address a similar field of analysis, this model reaches a more general form of solution whilst based upon a more compact set of precepts. It therefore has promise of both greater clarity and a broader basis for analysis and synthesis tasks in atoms, molecules and crystals. Once the principles of the model are grasped it promises relatively powerful visualization insights as well as a strong basis for computational analysis.

Though not part of its original remit, work on the structure of this model has brought into evidence its further potential for interesting and radical illumination of issues both nuclear and cosmological.

8.2 Proving Task

To make available the above virtues of this model it is now necessary to expand and detail its description, largely in areas of spin and the stochastic under-layer, and to check its computational performance and accuracy against the natural phenomena that it purports to represent. Descriptive accounts with examples of quantitative evaluation must be completed for each of a wide variety of physical phenomena. It is the purpose of this paper to lay down a self consistent set of principles upon which that rather compendious piece of work can then be based.

Were it to transpire that this model is less accurate than existing or future models in the physics that it predicts, still there is prospect that it could address its original objective in engineering work by virtue of its conceptual and computational compactness. In any case such a sense of limitation is no different from the position of any other model of those processes that are too complex for the human mind to bring into precise comprehension. It is made the more noticeable in the field of microscopic physics where a significant indeterminable attribute of the observer's own state is inherently combined into the system attributes being observed.

9 Acknowledgements

Text.

Appendices

Appendix A A Remarkable Smoothness

Within this form of wave model there are features that conspire to suppress the tendency for the mixing of wave components to produce up-converted frequency terms through intermodulation.

A.1 Freedom from Lagrangian Process Noise

The conventional approach to physics based upon primacy of the Lagrangian model of dynamics obscures this characteristic smoothness and consequently leaves an impression that the ultraviolet catastrophe is bound to occur. In the Lagrangian model the system is described in terms of alternating states of energy, changing between distinct forms like kinetic and potential energy. Having started with such "noisy" models this impression is then very hard to dispel by reasoning, a problem that is rampant in late twentieth century quantum electrodynamics.

In contrast we pursue here an Eulerian model of continuous complex fields in which the progress of state is a smooth advance of a phase with no essential fluctuation of magnitude. By building all observable interactions upon sesquilinear products of the wave variables the real and imaginary parts are not absolutely distinguishable ... only at most relatively. At every point in our complex fields there exists a plane in which the regularity of the Cauchy-Riemann differentiability exists. (Note: There is no corresponding elegant smoothness in the Lagrangian model unless it is enforced by combining two linked systems, and then there is no further point in using that kind of model.) Before observation can occur there is always a product of conjugate values to make a transformation into the domain of the real variable.

This smooth complex model is used in the simplest sufficient form compatible with observed reality. Nowhere in this model does the steady state of oscillation produce an inherent radiation of energy. There exists an equilibrium condition for electronic eigenmodes wherein, when superposed within a spin based atomic structure, they do not interchange or radiate energy. They can, however, absorb energy from the electromagnetic field to disturb that equilibrium, and radiation then ensues as fluorescence.

A.2 Modulation of Coefficients

So in electromagnetically quiescent conditions there is independence between electron modes within any spin based atomic structure. Nevertheless nonlinearity can still enter to produce fluctuations of excitation state without involving movement of the atoms because of the asymmetrical operation of the two differentiations in the quadratic form differential operator ... we are referring here to the coefficient matrix $\mathbf{K}$ in equation (3.7). The left hand differentiator operates on the product to its right, and thereby produces

coupling from fluctuations of the elements of the coefficient matrix. Thus the nonlinearity enters the system in a manner that is silent in the steady state, and carries only the lower frequencies appearing as Rabi fluctuations during perturbations of the energies of the system.

This confinement of the nonlinearity to the lower frequencies is a substantial assistance in making linear approximations in analysis of the system and the possibility of doing this appears to be unavailable in the conventional theories. It epitomizes the distinction between this kind of smooth continuum model and the classical collisional interaction systems with their tendency to up-conversion and therefore to ultraviolet catastrophe.

A.3 The Natural Suppression of Up-conversion

Perhaps the most remarkable sort of smoothness of dynamic properties in the model is the existence for oscillatory complex scalar waves in continuous space of a particular kind of intensity function for charge and current. This effect is represented by the Hilbert transform when used in a commutator structure.

The conventional form of intensity function for a complex scalar field is the squared norm, given by $y(t) = \psi^*(t).\psi(t)$ where ψ^* denotes the complex conjugate of ψ. Whilst this produces a constant intensity for any simple cisoid function, it produces a fluctuating value for any sum of cisoids with different frequencies ... consider the simple sum $\psi(t) = \mathrm{cis}(\omega_1 t) + \mathrm{cis}(\omega_2 t)$.

That squared norm intensity is the basis used by Schrödinger and also, slightly modified, by Dirac to derive charge density from the wave function. This kind of interaction then determines that any additive superpositions of electron wave functions would produce radiative up-conversion through fluctuations of charge and current densities. The conventional solution to this problem is to delay summing the terms until after using the intensity operator. To do that demands use of the high order so-called "Hilbert space" of system configuration to represent separately the multitude of electron states within atoms without invoking direct addition for their superposition. We too use a Hilbert space here as the coordinate parameter space of our additive waves, but it only ever has the usual 3 dimensions, not the 3N for each of two configuration parameters of position and momentum describing N Lagrangian particles.

The Hilbert transform offers an alternative kind of intensity or inner product that maintains the smoothness even when operating upon a sum of cisoids. Using this construct the summation of wave functions may be performed before evaluation of the intensities required for the charge and current densities.

Initially we justify the use of such a function here empirically ... it just appears best to correspond to reality. However there is a scheme that can generate this model in the statistics of an even simpler underlying stochastic structure. It offers unification of our de Broglie and Maxwell wave systems and acts as an interesting further intuitive justification for the form of this commutative operator. It also gets rid of the need for the temporally non-local average appearing in the structure of the Hilbert transform ... the only remaining nonlocality in the model used here. See further comment about this in appendix G.

The symbol $\boldsymbol{H}_t(.)$ denotes the Hilbert transform. This is an integral transform of the form:

$$\boldsymbol{H}_t(f(t)) \;=\; \textstyle\int_{-\infty}^{\infty} f(t+\tau)/\tau \; \mathrm{d}\tau \qquad t, \tau \in \mathbb{R};\; \boldsymbol{H}_t, f \in \mathbb{C}$$

By basing the model of energy density upon the squared norm of the time derivative of our wave function the corresponding real valued charge density can then be constructed from a commutator operation of the Hilbert transform on the wave function and its derivative:

$$y(t) \;=\; \psi^*(t).\boldsymbol{H}_t(\mathrm{d}\psi(t)/\mathrm{d}t) - \boldsymbol{H}_t(\psi^*(t)).\mathrm{d}\psi(t)/\mathrm{d}t \tag{A.1}$$

This can be applied directly to the charge scalar, and as a corresponding vector construct to produce current density ... see equation (2.1) above.

One way of viewing the smoothness is as a tendency to retain energy in oscillatory modes at lower frequencies, with minimal up-conversion leakages, to a degree that is greater than is possible in purely (mathematically) real systems. In these terms we may explain how the famous "ultraviolet catastrophe" of early twentieth century physics models does not occur in nature, and why bound atomic electrons in the steady state do not radiate.

A.4 Avoiding Dichotomous Duality

In particulate models an adequate result can only be achieved by incorporating a dichotomous duality of particles and waves. Note that the conventional wave/particle duality warrants this description "dichotomous" because it is not a full duality of alternative descriptions. It is a synthesis of parts necessary for use separately to accommodate otherwise unresolvable differences that are the consequence of the presumed existence of discrete entities and transitions. A symptom of this is seen in the peculiarly high orders of infinity arising in its Hilbert configuration state space descriptions. The continuum model discussed here does not need that, and so in at least that respect is less logically perplexing.

The objective here is therefore in a certain respect to explore and develop the especially smooth nature of interactions in the class of complex waves in a continuum so that models may be created that do not start with the disadvantage of implausibility at high frequencies. Where such models are applicable, and that appears to be the case in the complex variable wave dynamics of sub-quantal physical processes, the resulting approach is often simpler both to understand and to manipulate than are models based on conventional Lagrangian and event probabilistic ideas.

Appendix B Intrinsic Spin

B.1 The Necessity

The conventional model based upon the Schrödinger equation uses Coulomb potential to introduce structure via the non-differential part of the scalar field differential operator. In the spherical structure of an atom with point nucleus this provides a pole singularity of potential at the geometric centre and allows radial solutions in terms of Laguerre polynomials.

Our model is not based upon that Lagrangian potential concept but rather on the solution of wave equations driven essentially by the self generated charges and currents in the wave function solutions themselves. Electric field divergence rather than potential determines the topological structure here. It is because of this that unlike the Schrödinger model, this system contains the nonlinearity that can induce attractor states with quantal levels of charge in the solutions. The presence of a nuclear charge and its Coulomb field affects the stability and form of the electron field but not its topological structure nor the specific interval of the atomic charge quantisation levels.

This model does not require (indeed it requires the absence of) any such infinite central singularity to be involved in determining topology and quantal charge in the wave function solution. However a singularity with infinities in second and higher spatial derivatives does still occur at the central axial zero, at least because of the infinitesimal spatial wavelengths introduced near the axis by the azimuthal periodic harmonics. Nevertheless the wave function still meets the criterion of having finite energy ... which in this model corresponds to having finite integral of squared modulus temporal first derivative over the 3-space.

B.2 Spin Factor Structure

A central zero is needed, and there is no solution as a single eigenfunction that can provide that zero. Hence a composite factor is sought that is capable of doing that. The simplest solution found comprises four additive functions that when taken together are harmonic at every point except at the atomic axis where the function takes the value zero. The resulting matter waves display the required kinds of behaviour, in particular as relating to such as Stern-Gerlach spin test phenomena.

Considering $\psi(t, \mathbf{x})$ as the wave function of a single mode in an isolated atom in the steady state and using the polar co-ordinates r, ϕ, θ in place of the Cartesian basis $\mathbf{x}$:

$$\psi(t, \mathbf{x}) \;=\; \psi_P(t, r, \phi, \theta) \;=\; \kappa.\mathrm{cis}(m_l\theta).f_S(t, \theta).f_R(r, \phi) \tag{B.1}$$

... where we have:

$f_S(t, \theta)$	Temporal and azimuthal complex structure of the mode (normalized).
$f_R(r, \phi)$	Radial and colatitudinal real structure of the mode (normalized).
m_l	Azimuthal order of the mode (the associated magnetic quantum number).
κ	A complex multiplier of mode amplitude.

The amplitude normalizations of f_S and f_R are of no basic consequence since they can be compensated by corresponding changes to the constant κ and will anyway not affect the level of charge quantization as related to the unit wave function intensity at unit frequency.

The static structure of $f_R(r,\phi)$ is determined by the radial and latitudinal mode parameters of the particular electron mode considered. An offset is introduced to its structure by adding a differential operator term $\csc^2(\phi)/2$ that induces a radial factor $r^{0.5}$ into the solution. (It is incorporated in the spheroidal ripple equation (5.6).) That can then be compensated by the overall half order azimuthal complex twist in the $f_S(t,\theta)$ factor to leave no net effect on the overall wave equation. In this way the temporal-azimuthal factor $f_S(t,\theta)$ forming part of the spin factor can always be expressed in the form of four distinct but additive terms based on frequencies $\omega, 0, 0, -\omega$ (we may define this as having "unit amplitude co-spin mixture" form) by:

$$f_S(t,\theta) = \left[\begin{array}{c} \left[\begin{array}{l} \cos(\beta).\mathrm{cis}(\alpha+\theta) \\ +\jmath.\sin(\beta).\mathrm{cis}(\alpha-\theta) \end{array} \right] \cdot \left[\begin{array}{l} \mathrm{cis}((m_s+1/2)\omega t+\xi) \\ +\mathrm{cis}((m_s-1/2)\omega t+\xi) \end{array} \right] \\ +\mathrm{cis}((-m_s-1/2)\omega t-\xi) - \mathrm{cis}((-m_s+1/2)\omega t-\xi) \end{array} \right] /2 \qquad \text{(B.2)}$$

... where we have:

t	Time.
θ	Azimuthal angular coordinate.
ω	Modal angular frequency (sign characterized as in its twisted part).
m_s	Spin (anti) mode sense $= \pm 1/2$ (i.e. spin quantum number).
α	Twist (co-spin) azimuthal alignment angle.
β	Twist (co-spin) inverted superposition phasor.
ξ	Temporal sinusoid phase.

Figure B.1 gives a hint to its physical form in the sum of the two dynamic parts. There is azimuthal half twist of both phase and complex argument in the sum.

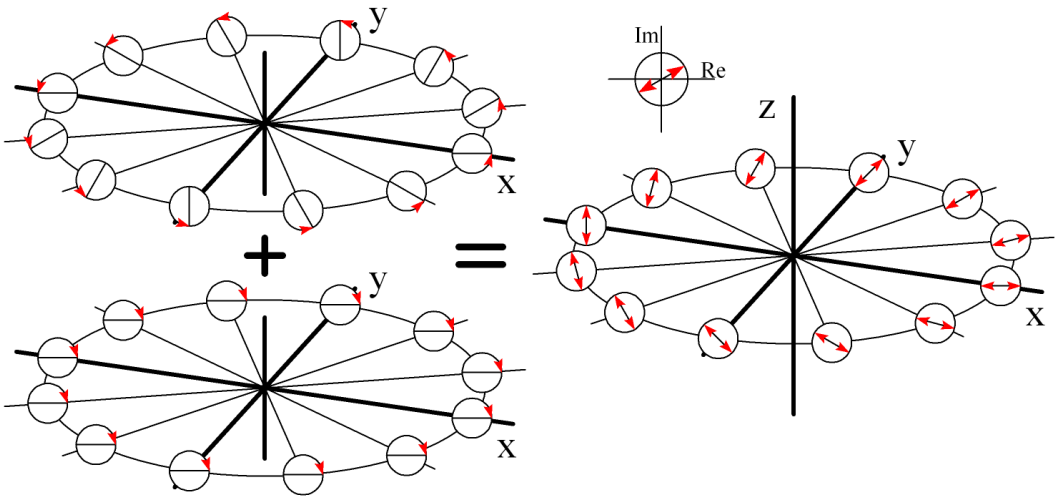

Figure B.1: **A Möbius Twist Built from Complex Components**

Paired "anti-spin" groupings of such "co-spin mixture pairs" can coexist in a range of superposition states with same values of the other quantum state indices.

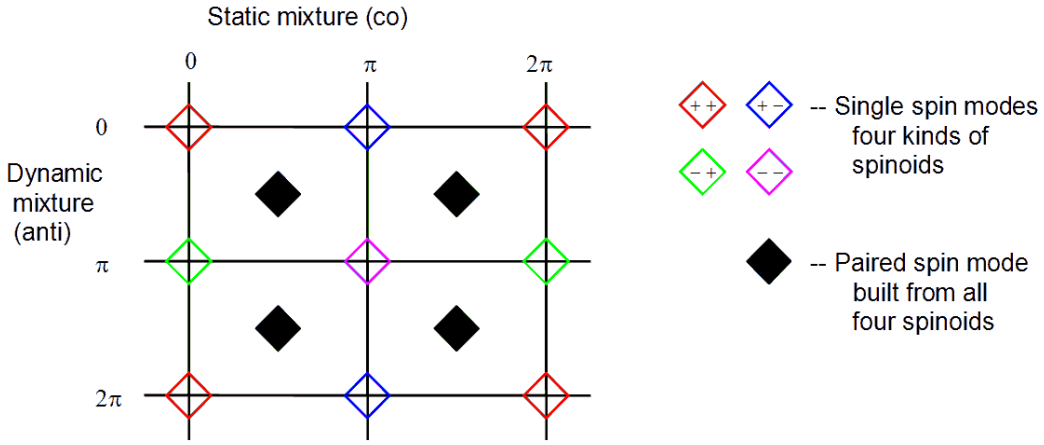

Figure B.2: **Kinds of Spin State Groupings**

Perturbations of the spin states in both the co and the anti directions on the above diagram are possible, including cases where the states gyrate periodically around the least energy positions that are shown on the diagram. Thus these states act as attractors in a variety of dynamic processes including those of the Stern-Gerlach phenomena that definitively characterise intrinsic spin.

Appendix C Existence of Electron Eigenmodes

C.1 Terms in the Mode-wave Differential Operator

The differential equation for any electron modal wave is of second derivative order in all four dimensions of space-time. Thus it contains a number of terms that for any eigen-mode must balance in each of the four dimensions at every point in that 4-space. We must study the set of possible waves that can satisfy this balance.

The curvatures of the spatial factor of any modal eigenfunction can vary in complicated ways over the three spatial dimensions, whereas the form of that same eigenmode in the time dimension must have identical frequency everywhere in space. Nothing else could remain continuous as is required for an eigenfunction. Only the phase of the temporal factor can vary over the spatial dimensions, not the frequency.

There is a constant term present in the operator. It is the basis of the dispersive nature of the matter waves ... they do not have constant speed as in the way we think of light. These matter waves always have phase velocity greater than, and group velocity less than that of light. This dispersion constant also determines the Compton frequency of the electron matter wave system.

The cubic term (that is responsible for the radial ripple structure) is separable from a residual linear term (as described in sections 4.2 and 4.3 above).

The presence of intrinsic spin introduces a constant term that is smaller in absolute value than and of opposite sign to the dispersion constant.

The magnetic terms act to increase the eigen-frequencies towards the Compton frequency, but are not significant to the existence of such eigen-modes. So they may be omitted from an account of existence alone.

C.2 The Finite Balance in Oscillation Frequency

The basic balance arises between the temporal frequency and the dispersion constant in the wave operator ... we may allow spatial curvatures of the function to be imagined sufficiently small as to be of lesser significance (a sufficiently flat wave) and so omit them from a first view. In differential terms, the positive dispersion constant in the operator is absorbed dominantly into the temporal dimension yielding the continual negative absolute curvature (towards zero) that manifests the oscillation that is necessary for persistence of matter. The bigger that negative constant, so the faster is the rate of oscillation. This basic picture determines the Compton frequency.

The intrinsic spin that is ever present in and essential to the bound state of matter contributes a positive absolute curvature (divergence) that acts to allow a reduction in the value of the negative absolute curvature (and thereby reduction of frequency) constituting the temporal oscillation. Without that reduction there would be no stable bound states possible.

The cubic term provides the equilibrium offset of modal eigen-frequency from the depressed Compton frequency ... there is nothing else can do that!

We can visualise this and thereby estimate quantitatively where the balance occurs in terms of the eigen-frequencies produced.

Appendix D Inter-Modal Couplings

This appendix is in urgent need of substantial correction. Omit it for the time being.

D.1 Local Interaction Structure

D.2 Precessions

Appendix E Coefficient Matrix Structure

E.1 Unitarity of the Coefficient Matrix

There are structural requirements of local electromagnetic field relationships to be met at each point in the electron field in order for steady state stability of the electron modes to be possible. Conditions for steady temporal oscillatory operation of any mode are that the magnitude at every point shall remain constant in time and the pattern of relative phase in the mode shall remain constant in time, requiring precise equality of frequency everywhere. It falls to the overall form of the differential operator to comply with and maintain these conditions and that is only possible under stable approach to an attractor state.

We may use equivalently either a Klein-Gordon or a Helmholtz form of equation to represent the system we are describing, and the conditions appear in slightly different forms in the two respective cases.

In the Klein-Gordon format the differential operator must be unitary with negative unit determinant. The off diagonal elements of the matrix coefficient of the generalised d'Alembertian have antiderivative nature, so to avoid the difficulty of handling this we use here its transformation into the Helmholtz form of equation.

In the Helmholtz format the differential operator must be normal, and after subtraction of the conjugate quadratic eigenvalue from the leading element must leave only a hybrid transformed generalised d'Alembertian part with 4×4 coefficient matrix that is unitary, and still with negative unit determinant. It is this need for unitarity in the presence of the (relatively small) electromagnetic off-diagonal coefficients that demands small compensating perturbations added to the simple d'Alembertian diagonal.

Refer for details of normal and unitary operators and their associated matrices to the mathematics of "The Spectral Theorem" and perhaps also C*-algebras.

In a state near to a stable attractor the shape of each modal wave function and the value of its respective eigen-frequency can self adjust to approach this balance in the electromagnetic field, and for precise results the form of the matrix used in our analysis must likewise be adjusted to bring about this condition.

Fortunately for most purposes the corrections to achieve normality are small, and may be studied as an additive perturbation to the four diagonal elements of the matrix. We may as follows deduce the correction to the equation in its matrix modulated Helmholtz form EMHM ... the hybrid frequency domain form. Because the frequency term ω_n^2 in this equation is separated from the matrix the result expressed in that matrix applies simultaneously to all modal frequencies.

Thus the condition of normality of the Helmholtz 4×4 hybrid differential operator corresponds to unitarity of the frequency independent matrix. However, the result depends upon the specific charge density profile of the electron field concerned, so iterative method is likely to be needed for accurate evaluations.

Recapping equation EMHM (3.7) and its coefficient matrix:

EMHM

$$\left[\left(\Diamond_1^T \cdot \mathbf{K}(s,\mathbf{x})\right) \star \Diamond_1 + \omega_n^2\right] \Psi_n(s,\mathbf{x}) \;=\; 0 \tag{E.1}$$

... where:

$$\mathbf{K}(s,\mathbf{x}) \;=\; \mathbf{M}\delta_\omega + \mathbf{F} + \mathbf{W} \;=\; \begin{bmatrix} W_{00}-\delta_\omega & -E_1 & -E_2 & -E_3 \\ E_1 & W_{11}+\delta_\omega & \jmath U_3 & -\jmath U_2 \\ E_2 & -\jmath U_3 & W_{22}+\delta_\omega & \jmath U_1 \\ E_3 & \jmath U_2 & -\jmath U_1 & W_{33}+\delta_\omega \end{bmatrix} (s,\mathbf{x})$$

... and $\mathbf{W}$ is the perturbation of the diagonal in which we are interested, consisting of elements $(W_{00}, W_{11}, W_{22}, W_{33})$.

The $\star$ symbol denotes convolution and the symbol δ_ω represents a unit impulse function of the temporal frequency ω.

For this coefficient matrix operating in equation EMHM (E.1) to produce an eigenmodal solution it must yield a uniformly coherent eigenvalue throughout $\mathbf{x}$. Considering the long waves of modes approaching the Compton frequency it must also satisfy approach to the zero electromagnetic field case (Minkowski unit diagonal alone). The matrix must therefore always be (pseudo-)unitary because only then can the differential operator be normal and the solution persist in a steady oscillatory state without inward or outward radial transfer of energy (though circulation is permitted).

To achieve such a unitary matrix there are consequent row and column norm requirements needing $\mathbf{w}$ to comply as follows:

$$\|\mathbf{e}\|^2 \;=\; 2w_{00} - w_{00}^2 \quad \text{and} \quad e_k^2 + v_k^2 - \|\mathbf{v}\|^2 \;=\; 2w_{kk} + w_{kk}^2 \quad 1 \le k \le 3 \tag{E.2}$$

The following approximation is for $|w_{kk}| \ll 1$ (where this unit value corresponds to the square of the Compton frequency in our normalized units). The quadratic terms are usually negligible when handling electrodynamics so the following approximation is then useful.

$$w_{00} \;\approx\; \|\mathbf{e}\|^2/2 \quad \text{and} \quad w_{kk} \;\approx\; (e_k^2 + v_k^2 - \|\mathbf{v}\|^2)/2 \quad 1 \le k \le 3 \tag{E.3}$$

Define the parts of the diagonal perturbation as a scalar $w_e = w_{00}$ and a vector $\mathbf{w}_m = (w_{11}, w_{22}, w_{33})$. The presence of a perturbation in the diagonal of the coefficient matrix gives rise to a further term in the expanded version of the electron modal equation (E.1), and it includes a part induced by the divergence of $\mathbf{w}_m$. However for most purposes, and particularly for the outer electrons of lighter elements, this latter term is small.

E.2 Finding Stable Solutions

To study the stability of atomic processes we may question how the interplay between the matter wave system and the electromagnetic field finds an equilibrium.

According to the above normalising rules (E.2) and (E.3) the self generated electromagnetic field of an electron wave localised in a given region must act everywhere in that region to increase the magnitude in the elements of the matrix diagonal. Put another

way, in an electromagnetic field the determinant of the coupling matrix is always driven negative from the free space value of -1. This is caused at least through the perturbation terms quadratic in w_{00} due to the electric field (see equation (E.2)), and additionally by any magnetic field present (terms linear in w_{kk} even more clear from equation (E.3)). The unitarity of the differential operator is then recovered by an increase in the eigenfrequency of each mode. The effects of terms linear in w_{00} arising from the electric field are seen to cancel overall.

Unitary operator solutions can be found in which this augmentation of the matrix diagonal elements is offset by a modal frequency higher than the Compton frequency. Such is the case for a free electron wave, and we may see that it can operate without a central singularity or any special processes like intrinsic spin. It provides a model under which to study the dynamics of ejection of an electron to the free state in the ultraviolet ionisation of an atomic gas. But this simple kind of compensation by frequency increase is not enough by itself to yield solutions corresponding to bound electron states with their lower modal frequencies.

For bound electron waves the modal frequencies are less than the Compton frequency. Their existence requires overall unitarity of the matter wave equation differential operator, and for steady oscillatory state these solutions require the presence of intrinsic spin structure. That in turn can only operate in the presence of a centrally located nucleus with positive charge to generate a stabilising radial Coulomb field. How that original stable positive charge is manifested in the nucleus will not be dealt with here.

E.3 Universal Diamagnetism

The application of a magnetic field from an external source produces a net augmentation of the matrix diagonal elements, so can be compensated by an increase in the frequency of every electron mode present. Because the effect is strongest where the magnetic field is strongest, it will induce a phase twist in each electron mode with phase velocity directed from stronger to weaker magnetic field. That inherently brings a corresponding accelerative tendency in the electron wave group away from stronger magnetic fields.

This is a basis of explanation and analysis of a commonplace form of the magnetic effect called diamagnetism. This is a universal low level effect present in all matter (famously sufficient to levitate a frog in a strong magnet field), but it is small by comparison with other dia and paramagnetic effects that arise in certain cases from various structures within particular species of atoms, molecules and crystals. (Consider paramagnetic oxygen O_2 gas.)

These diamagnetic effects are linear in the magnetically induced perturbations of the matrix diagonal, but, according to the above formulæ, with a further practically much weaker term they are quadratic in the perturbation in opposition (paramagnetic) as well.

There is no para or dia effect linear in the electric perturbation of the matrix, but these equations indicate that there remains an extremely weak dia-electric effect in the quadratic perturbation.

Appendix F Wherefore Discreteness?

F.1 BKS Debate - the Photon

There was a battle of ideas in the 1920s involving Einstein, Born, Heisenberg, Planck, Bohr-Kramer-Slater (BKS theory), and others. Notwithstanding their conclusion, we here approach on a different basis, namely that we must avoid treating elemental particles bearing mass as fundamental. They must instead emerge under a model based directly on causation amongst field amplitudes. The quadratic (sesquilinear) products arising in this model then produce these material properties of mass and charge and the associated manifestations of momentum and force.

Discuss here electron transit in crystals ranging from conduction in metals to excitation of high order eigenmodes. Use this to account, at least in outline, for the photoelectric and Compton effects.

Going further, the model uses an assertion that, with sufficient care in the details, all EM radiation from matter can properly be treated as continuous, covering electronic fluorescence up to X-ray frequencies, as well as nuclear gamma radiations. This usually produces exponential decay of radiated intensity from individual coherent atomic or poly-atomic processes, whilst excitation can arise either progressively from incident radiation or else as the result of abrupt attractor (e.g. ionic) state changes of atoms leading to their subsequent equilibrating radiation.

This approach drives the account of quantum event based behaviour squarely into the domain of matter state changes, not into the structure of coupling fields mediating interactions of matter. This is valuable because it is in matter that the processes of discrete state attractors can be causally explained without further specific precepts ... as the model proceeds to do.

In particular this model asserts the necessary involvement of discrete state changes of matter in all processes of amplification (perhaps yet barring some arcane exceptions) and that such amplification is essential in microscopic observation. The destruction of information inherent in such discreteness in amplification then accounts for the ambivalence in observed data, bringing forth the notions of “complementarity”.

F.2 Complementarity

Over this issue of complementarity, the position taken in using the form of model developed here acknowledges that microscopic physical observations will always be subject to the limitations imposed by the need for amplification. But that does not limit the principle of deterministic precision expressible to represent the mechanism underlying such observable phenomena.

The example of Fourier transform complementarity exemplifies this. Whereas expression in one form limits the information scope of its transform, that is not due to any underlying imprecision, only to the way we choose to look at things.

Another kind of argument applies, in a more existential way, to the pairs of conjugate variables involved to produce the product variables that are the only possible form of observables. (Add regard here to QM concepts of "an observable" and the rôle of "eigenstates".) This limitation on observability does not invalidate using precise and deterministically expressed structure to explain the form of mechanism that underlies those conjugate variables. What might appear a redundant description of two variables regarding any particular observation is not redundant if it makes the model capable of leading to the correct averages over many observations.

Thus the uncertainty principle emerges as in quantum mechanics, being a barrier to absolute precision in each observation. But it does not limit the use of precision in potential underlying models, nor does it necessarily destroy value in the use of such precision of hypothetical expression, so long as the application of such a model makes apparent this unavoidable uncertainty in individual observations. It does not even destroy the possibility that nature might work in just such a precisely determined way. It just means that the human state, though capable of rather precise knowledge in general, can never be privy to precision in the indefinite detail.

Averages taken over indefinitely large samples of tests are not limited by the principle of uncertainty in observation. Thus so long as the application of the model produces those same averages there is no need to impugn its causal or determinist precision. The virtue of the model over this issue then lies in its comprehensibility ... an aspect in which current quantum mechanical models centred on ideas of probability as the ultimate basis appear less than ideal.

Appendix G A Meta-model

Careful completion of this appendix G is in process.

G.1 Stochastic Substrate Processes

The structure of the matrix coupling in the main equation EKG (3.1) or in its frequency domain equivalent EMHM (3.7) might be accepted on an empirical basis ... merely based upon tests that it fits with observed physical phenomena. To do things that way was broadly the objective in this paper and also, originally, of the project. However, there arise puzzles with the Hilbert transform structure in equation EHt (2.1) in the charge and current origination part of the model and also in trying to account for the phenomena of the Fermi distribution and Pauli exclusion. From these challenges there arises a sense that a noise based field correlation process may be able to produce the required structure in its statistical field parameters to account for these phenomena. Such an extension into a supporting under-layer offers an improvement in the general conviction carried by the model.

The feasibility of such a stochastic under-layer does indeed appear possible and it does address the issues mentioned. However at the time of this writing (16 March 2014) it is still under development, so cannot be given as a direct reference for this paper. Still it is perhaps valuable to give an idea of how the coupling matrix structure can come about in terms of such a deeper model.

So given below are some hints at the overall behavioural nature of this kind of stochastic system and its ability to unify disparate parts of physics. Beyond a brief mention, the reasoning and maths necessary to clarify and justify this assertion will have to wait for separate presentation after completion of some further work on the subject.

This stochastic under-system rests upon properties of the correlations between a substrate complex scalar noise field and its gradient vector, and also between the elements of that gradient. When such a correlation is developed there are real terms in self interaction of noise in the diagonal of the matrix, and there are inter-dimensional interaction terms that are imaginary and that appear in the off diagonal elements. This combination of strong real terms arising in the diagonal and weak imaginary terms off the diagonal has structure at least similar to that indicated in equation EMHM (3.7). Hence we see its initially apparent relevance.

G.2 The Substrate Scattering Interaction

Theory of thermodynamic interactions tells us how equipartition of energy leads to fluctuations over a Gaussian distribution of amplitudes in the respective individual orthogonal degrees of freedom of motion in the system. In contradistinction to this the complex scattering interaction in the substrate field considered here is based upon a different kind of interaction. This produces a distribution in each orthogonal resonant system with proportionately far narrower range of fluctuation about its mean value, and also with a tendency

to concentrate energy into lower frequencies in a manner unlike the equipartition of thermodynamics. In this process there is still a tendency to equipartition, but it occurs not in the energy but in the charge, and is confined to frequencies below a cut-off point. Briefly it operates as follows.

We consider a simple scalar wave system on the four dimensional Minkowski space-time. It can be expressed as conforming to the d'Alembertain wave equation $\Box p = 0$ where p is complex scalar. When this system is driven by incident wide band noise (arriving from the cosmos) it is capable of supporting certain forms of localised bi-cisoid oscillatory soliton structures. (See section 2.1 above for "bi-cisoid".)

Such a soliton scatters the noise, and if the incident noise has spectral kurtosis $k > 0$ (it is leptokurtic) then the soliton can self-sustain by virtue of kurtosis reduction (i.e. the tendency toward statistical equilibrium) in the scattered wave field. (Note that to mix for kurtosis reduction the bi-modal spectral line structure of the bi-cisoid soliton process is the ideal platykurtic spectrum having $k = -3$.)

The resonant excitation of that soliton will then tend to an intensity that is related to both the value of the excess kurtosis and the resonant frequency of the soliton. The frequency of the soliton will also be related to the value of the kurtosis, but in a way that depends additionally upon the wave structure of the soliton. We seek to find the equilibrium intensity induced in each independent coherent resonator by the active (i.e. having excess kurtosis) substrate field.

G.3 Rationale for Charge/Current Origination

For charge and current to be generated as a local function of ψ-wave and its gradient requires that it shall be produced as the conjugate product of the amplitude and the temporal Hilbert transform of its 4-gradient. Only this way can it develop the Planck constant frequency proportionality as required whilst also producing the same sign of charge over both signs of frequency.

This is equivalent to saying that the charge and current are generated by the quadratic (sesquilinear) self products of the sum of the individual modal terms present. Interactions of the fields of the individual electrons then produce only the multi-pole patterns with zero mean that are the source of spontaneous emission. The black body Planck distribution of electromagnetic radiation so produced is in thermodynamic balance with the Fermi distribution of the electron excitation levels.

For the latter case, and also in order to support the first degree effect in frequency (Planck's constant), the scattering mode frequency must remain evident in the scattered substrate (Check this!). So that then forms the basis of the substrate stochastic model.

Even if we rely upon such linear scattering and post propagation sesquilinear mixing, we still need to show that the influences of the electromagnetic field generated in this way carry no terms of up-converting intermodulation between the original scattering wave modes.

If the first moments of cross spectra can generate the required effects at the point of influence we may be through without any inter-modal up-conversion terms. That is possible

because the spectra are sesquilinear in the scattered field which is in turn linear in both the modal waves and the original substrate. Because the scattered field has its substrate "carrier" terms incoherent across the Planck band there are no cross spectral terms in the modal frequency differences ... in other words, there can be no algebraic addition of original basis terms of the modal waves once they are represented in cross spectra of the scattered substrate field. Maths proof or at least expression needed here.

As a further feature of this scheme we might look to the inherent symmetry of frequency sign implicit in the individual modal waves being always bicisoid in any sufficiently small region. This necessity brings with it a vindication that the charge/current generation shall by some means be dependent upon modulus of frequency.

The first moment of cross spectrum does indeed have this feature of non dependence upon sign of originating wave frequency. Thus so long as the influences of electromagnetic field can be shown to be correct when based upon these first moment cross spectra then the inter-modal interference effects are suppressed as required.

We may summarize by saying that the process of splitting the influence of the modal waves into cross spectra in the scattered substrate leads to influences that carry no cross modal terms. They are equivalent to the spectral terms that might be obtained from the original modes by overall sesquilinear self product with a long duration of averaging. But these effects are instantaneous to within periods short compared to the Compton frequency. They are formed by averaging over a duration that is long only by comparison with the Planck frequency. The price paid for this fast process of spectral formation is a small noise resulting from averaging the substrate field over its full bandwidth and appearing as zero point fluctuation.

It is planned that further clarification of this structure will be forthcoming in another essay dealing with the stochastic basis of the model.

G.4 Pauli Exclusion and the Fermi Distribution

The overall charge associated with any given spin axis is maintained quantal by the basic ripple process ... see Eripple (4.10) ... and does so to within fine limits and at high frequency. Still the equilibrium with the stochastic substrate intensity at its given kurtosis determines the limit of charge generated within any coherent mode. At the limit of zero absolute temperature that charge value is achieved by each of the number of electron modes equal to the discrete charge of the electronic system. Each electronic charge is then the same, regardless of the frequency of its respective modal resonance.

At temperatures above absolute zero, thermal processes communicated by the electromagnetic field then act to couple between electron modes, and to do so more strongly between those with smaller differences of resonant frequency. This produces the characteristic form of the Fermi distribution for the pattern of equilibrium excitation levels of the individual electron modes.

Appendix H "Azimuthal" Confusion

Ambiguous nomenclature – Take care!

There is an unfortunate clash of nomenclature regarding the term "azimuthal" as inherited from the conventions of twentieth century quantum mechanics. In talking of spherical systems with an axis it is proper to use the adjective "azimuthal" to describe patterns, positions and angles in the equatorial plane of the system. In pursuits such as gun aiming and also in wave systems such as are described in antennas and resonators this convention is the one used. Thus the azimuthal order of a spherical harmonic is most naturally associated with the number of cycles of the variable occurring in a circuit around the axis. The harmonic order constituted as the number of half cycles of the variable along the latitude angle from pole to pole would then tend to be referred to as the latitudinal or possibly the axial order. The remaining spherical (ball) order is then the radial, being the number of half cycles in the radial form from centre to periphery.

Unfortunately because of the early observation of magnetic quantization in isolation the term "magnetic quantum number" denoted by m_l became associated with what transpires in a wave model to be what we would call the azimuthal order of the wave mode. We might as well retain that symbol for it.

Meanwhile, as a result of spinning top momentum models the term "azimuthal quantum number" (and even more confusingly occasionally "orbital quantum number") denoted by ℓ (letter L) then became associated in a rather complicated way with what in a wave model would be called the latitudinal (or possibly axial) harmonic order of the wave mode. Here we define the latitudinal (axial) harmonic order as the number of half waves along the entire colatitudinal pole to pole angle (i.e. π) and denote it as l_c. It can take values from 1 to n where n is the principal quantum number which denotes the given energy group (chemical period). This latitudinal (axial) order can be defined in terms of the conventional quantum numbers as $l_c = n - \ell$.

The radial order becomes equal to the principal quantum number when the other two orders are at their minimum absolute values $l_c = 1$ and $m_l = 0$. As a harmonic structure the radial order (i.e. number of radial half cycles) is therefore equal to $n - l_c - |m_l| + 1$, which would be $\ell - |m_l| + 1$ in the earlier convention.

References

[AA1982] A.Aspect, J.Dalibard, G.Roger *Experimental Test of Bell's Inequalities Using Time-Varying Analyzers*
Phys.Rev.Let. v.49 n.25 20 Dec 1982 pp.1804-1807

[AB1959] Y.Aharonov & D.Bohm *Significance of Electromagnetic Potentials in the Quantum Theory*
Phys. Rev. 115, pp.485 - 491 (1959)

[AB1961] Y.Aharonov & D.Bohm *Further Considerations on Electromagnetic Potentials in the Quantum Theory*
Phys. Rev. 123, pp.1511 - 1524 (1961)

[AB2004] A.G. Booth *Essential Structure in Physical Observation*
Kybernetes Vol.33 No.9/10, 2004, pp. 1419-1432
Related essay is also available at http://wavemodel.org/obs/observ.html

[AE1975] L.Allen & J.H.Eberly *Optical Resonance and Two Level Atoms*
Dover 1987 ISBN 0-19-509345-3 Originally published by Wiley, New York 1975.

[AE1935] A.Einstein, B.Podolsky, N.Rosen *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*
Physical Review v.47 May 15, 1935 pp.777-780

[AG2004] Alex Granik *Schroedinger revisited: How the time-dependent wave equation follows from the Hamilton-Jacobi equation* arXiv:quant-ph/0409018v2 6 Sep 2004

[AG2006] A.Gover *Spin-polarized Free Electron Beam Interaction with Radiation and Superradiant Spin-flip Radiative Emission* PhysRevSTAB. **9**.060703 (2006)

[AG2008] A.Gsponer *What is spin?* arXiv:physics/0308027v3 10 Sep 2003

[AS2003] Alwyn Scott *Nonlinear Science - 2nd Edition* Oxford 2003

[AU2000] Alexander Unzicker *What can Physics learn from Continuum Mechanics ?*
arXiv:gr-qc/0011064v1 20 Nov 2000

[AZ1999] A.Zeilinger *A Foundational Principle for Quantum Mechanics*
Foundations of Physics, v.29 n.4 1999 pp.631-643

[BG1999] B.M.Garraway and S.Stenholm *Observing the Spin of a Free Electron*
Phys.Rev.A v**60** n1, 63-79 (1999)

[CF1977] E.Cartmell & G.W.A.Fowles *Valency and Molecular Structure - 4th edition*
Butterworths 1977

[EC2004] E.Comay *Difficulties with the Klein-Gordon Equation*
Apeiron, v.11, n.3, Jul 2004 http://redshift.vif.com/Apeiron_Home.htm

[EC2005] E.Comay *Further Difficulties with the Klein-Gordon Equation*
Apeiron, v.12, n.1, Jan 2005 http://redshift.vif.com/Apeiron_Home.htm

[EC2006] E.Comay *Theoretical Errors in Contemporary Physics*
Apeiron, v.13, n.2, Apr 2006 http://redshift.vif.com/Apeiron_Home.htm

[ES1933] Erwin Schrödinger *The Fundamental Idea of Wave Mechanics*
Nobel Lecture, December 12, 1933

[FD2007] Fu-Guo Denga, Xi-Han Li, et al *Economical Quantum Secure Direct Communication Network with Single Photons*
arXiv:quant-ph/0606008v2 13 Dec 2007

[GG2006] G.Gabrielse, D.Hanneke, T.Kinoshita, M.Nio, B.Odom *New Determination of the Fine Structure Constant from the Electron g Value and QED*
Physical Review Letters v97, 030802 (2006)

[GG2006a] Gerald Gabrielse, *The Magnet in the Electron* Physics World, Feb 2007, pp.32-36

[GK2007] M.Gondran, A.Kenoufi *Complex Faraday's Tensor for the Born-Infeld Theory*
arXiv:0708.0547 v1 [math-ph] 3 Aug 2007

[JB1964] J.S.Bell *On the Einstein Podolsky Rosen Paradox*
Physics v.1 n.3 pp.195-200 1964

[JB1987] J.S.Bell *Speakable and Unspeakable in Quantum Mechanics*
Cambridge 1987 ISBN 0-521-36869-3

[JB2000] J.Bastero, M.Milman & F.J.Ruiz *Commutators for the Maximal and Sharp Functions*
Proc.AMS v.128 n.11 pp.3329-3334 (Electronically published on May 11, 2000)

[JM1996] J.Moret-Bailly *Quantum Mechanics an Approximation of Classical, Non-Linear Physics?*
Apeiron v.3 n.2 pp.27-30 (April 1996)

[KC2006] Klaus Capelle *A Birds-Eye View of Density-Functional Theory*
arXiv:cond-mat/0211443v5 18 Nov 2006

[LdB1939] Louis de Broglie, *Matter and Light* W.W.Norton - New York 1939

[LdB1987] Louis de Broglie, *Interpretation of quantum mechanics by the double solution theory*
Annales de la Fondation Louis de Broglie, Volume 12, no.4, pp.1-23 1987

[NL2004] N.Lütkenhaus *Quantum Key Distribution - How Do We Know It's Secure?*
Optics & Photonics News, March 2004 pp.24-28

[OB1956] O.Buneman *Continuum electrodynamics and its quantization*
Il Nuovo Cimento Vol 4, Supplement 2 / April, 1956 pp.832-835

[OB1997] O.Blasco & P.Villarroya *Commutators of Linear and Bilinear Hilbert Transforms*
Proc.AMS v.?? n.?? (1997)

[PO2003] Paul O'Hara *Rotational invariance and the spin-statistics theorem*
arXiv:quant-ph/0310016v1 3 Oct 2003

[PW1998] P.J. Werbos *New Approaches to Soliton Quantization and Existence for Particle Physics*
arXiv:patt-sol/9804003v1 8 Apr 1998

[PW1999] P.J. Werbos *Can Soliton Attractors Exist in Realistic 3+1-D Conservative Systems?*
Chaos, Solitons, & Fractals. November 1999, v.10, n.11, pp.1917-1946

[RH2004] Rom Harre *Modeling: Gateway to the Unknown*
Vol. 1, Ed. Daniel Rothbart, Elsevir 2004

[SW2002] S.M.H. Wong *What exactly is a Skyrmion?*
arXiv:hep-ph/0202250v2 28 Feb 2002

[TK2001] T.Kinoshita, *Improvement of the fine-structure constant obtained from the electron $g-2$.*
IEEE Transactions on Instrumentation and Measurement. April 2001 Volume 50, Issue 2, pp.568-571

[TM2002] Trevor W.Marshall, *Nonlocality - The party may be over*
arXiv:quant-ph/0203042v1 9 Mar 2002

[TMa1997] Trevor W.Marshall, Emilio Santos, *The myth of the photon*
arXiv:quant-ph/9711046v1 20 Nov 1997

[VB1992] V.B.Braginsky & F.Y.Khalili *Quantum Measurement*
Cambridge University Press, (1992)